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Fuzzy Linear Labeling

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Abstract **- A fuzzy linear labeling of a graph is the existence of two injections which assigns values in [0,1]. This labeling fuzzifies the crisp graph. In this paper we show the admissibility of fuzzy linear labeling in some path related graphs.**
Keywords: **Path Graph, Square Graph, Duplicate Graph, H-Graph, $P_n \odot K_1$.**
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I. INTRODUCTION

All graphs we consider in this paper are all finite, simple and undirected. The first definition of fuzzy graph by Kaufmann was based on Zadeh's fuzzy relations. Rosenfield considered fuzzy relation on fuzzy sets and developed the structure of fuzzy graphs. After the work of Rosenfield, Yeh and Bang introduced various connectedness concepts of graphs and digraphs into fuzzy graphs. In [1], A. Nagoor Gani and D. Rajalaxmi (a) Subahashini introduced the concept of fuzzy labeling.

A graph is said to be a fuzzy labeling graph if a fuzzy labeling is defined on it. A fuzzy graph G = (V, m, ρ) is a non-empty set V together with a pair of functions, m : V →[0,1] and ρ : V x V →[0,1] such that for all x, y in V , ρ(x, y) ≤ m(x) Λ m(y),where m is the fuzzy vertex set of G and ρ is the fuzzy edge set of G respectively. In this paper we identify some path related graphs that admits fuzzy linear labeling and proves the admissibility.

II. MAIN RESULTS

Definition 2.1

Let G = (V,E) be a crisp graph, where V is the ordered set of vertices and E is the set of ordered pair of edges. Then G has a fuzzy linear labeling if there exist two injections m and ρ such that m:V→[0,1] defined by m(v_i) = − , where V=($v_1, v_{2,\dots,} v_N$), $v_i \in V$ and ρ: E→[0,1] defined by

) = ———,

The graph G which admits fuzzy linear labeling is called a fuzzy linear labeled graph.

Example 2.2

Consider the path graph P_5, where $|V$)| = 5,$|E$)|= 4.
Let V =($v_1,v_2,\dots,v_5$) and

Define two functions m: V→[0,1] and ρ: E →[0,1] as follows:

m(v_i) =− , 1 and ρ(v_i, v_{i+1}) =———
——, 1 4.

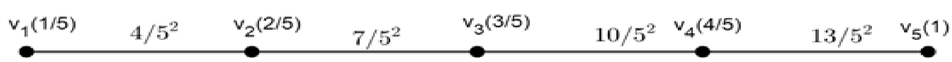

Fig. 1Fuzzy Linear Labeled Graph

Theorem 2.3

Every fuzzy linear labeled graph is a fuzzy graph.

Proof

From the definition of fuzzy linear labeling,

) =——— m(v_i) Λ m(v_j) for all i and j with i < j.

Theorem 2.4

Fuzzy linear labeling is a fuzzy labeling if and only if j (mod N-2) for all i, j with i < j.

Proof

By definition, : E→[0,1] is said to be fuzzy labeling ifρ) < m(v_i) Λ m(v_j) for all i and j.

If possible, suppose that) = m(v_i) Λ m(v_j) for some i and j such that i < j.

This means thatρ) = m(v_i) ——— =m(v_i).
2i+j= i N ⟺j).

Result 2.5

Two fuzzy linear labeled graphs are said to be equal if their underlying graphs are isomorphic.

Note 2.6

For a fixed number of vertices, fuzzy linear labeled graph is unique.

Definition 2.7

A vertex v is said to be a common vertex in two fuzzy linear labeled graphs G_1 and G_2 if
G_1 and G_2 have the same number of vertices and $m_1(v) = m_2(v)$.

An edge e is said to be a common edge of two fuzzy linear labeled graphs G_1 and G_2 if G_1 and G_2 have the same number of vertices and $\rho_1(e) = \rho_2(e)$.

Result 2.8

The order of a fuzzy linear labeled graph with N vertices (N+1)/2.

Proof

Order = $\sum_{i=1}^{N} m(v_i)$ =(1+2+...+N)/N =(N+1)/2.

III. ON PATH RELATED GRAPHS

In this section we show that the square graph of path, duplicate graph of path,H-graph of path and $P_n \odot K_1$ admit fuzzy linear labeling.

Theorem 3.1

The square of path P_n^2 admits fuzzy linear labeling.

Proof

Consider the square of P_n, P_n^2= (V,E) where $|V|$ = n and $|E|$=2n-3
Let V =($v_1, v_2, \ldots, v_n$).

Define two functions m: V →[0,1] and ρ: E →[0,1] as follows:
$m(v_i) = \frac{i}{n}$, $1 \le i \le n$ and
ρ: E →[0,1] as $\rho(v_i, v_{i+1}) = \frac{3i+1}{n^2}$, $1 \le i \le n-1$.
$\rho(v_i, v_{i+2}) = \frac{3i+2}{n^2}$, $1 \le i \le n-2$.

If possible, suppose that m is not injective.
Then there exist some i, j such that $m(v_i) = m(v_j) \Longrightarrow$
$\frac{i}{n} = \frac{j}{n} \Longrightarrow$ i =j

Also if ρ is not injective, then there exist some i,j such that
1. $\rho(v_i, v_{i+1}) = \rho(v_j, v_{j+1})$
$\Longrightarrow \frac{3i+1}{(2n-1)^2} = \frac{3j+1}{(2n-1)^2} \Longrightarrow$ i = j for $1 \le i \le n-1$.
2. $\rho(v_i, v_{i+2}) = \rho(v_j, v_{j+2})$
$\Longrightarrow \frac{6i+2}{(2n-1)^2} = \frac{6j+2}{(2n-1)^2} \Longrightarrow$ i = j for $1 \le i \le n-2$.
3. $\rho(v_i, v_{i+1}) = \rho(v_j, v_{j+2})$
$\Longrightarrow \frac{6i+1}{(2n-1)^2} = \frac{6j+2}{(2n-1)^2}$ for $1 \le i \le n-1$ and for $1 \le j \le n-2 \Longrightarrow 6(i-j) = 1$, not possible since i and j are natural numbers.

So m and ρ are injective.

Theorem 3.2

The duplicate graph of path graph P_n, admits fuzzy linear labeling.

Proof

Consider the duplicate graphDP_n =(V,E) of P_n, where $|V|$ = 2n,$|E|$= 2n-2.
Let V =($v_1, v_2, \ldots, v_n, u_1, u_2, \ldots u_n$).

Define two functions m: V→[0,1] and ρ: E →[0,1] as follows:
$m(v_i) = \frac{i}{2n}$, $m(u_i) = \frac{i+n}{2n}$, $1 \le i \le n$ and
ρ: E→[0,1] as $\rho(v_i, u_{i+1}) = \frac{3i+n+1}{(2n)^2}$, $1 \le i \le n-1$.
$\rho(v_i, u_{i-1}) = \frac{3i-1+n}{(2n)^2}$, $2 \le i \le n$.

If possible, suppose that m is not injective.

Then there exist some i,j such that
1. $m(v_i) = m(v_j) \Leftrightarrow \frac{i}{2n} = \frac{j}{2n} \Longrightarrow$ i =j.
2. $m(u_i) = m(u_j) \Leftrightarrow \frac{i+n}{2n} = \frac{j+n}{2n} \Longrightarrow$ i =j.
3. $m(v_i) = m(u_j) \Leftrightarrow \frac{i}{2n} = \frac{j+n}{2n} \Longrightarrow$ i-j = n, not possible.

Also if ρ is not injective, then there exist some i, j $\le n$ such that

1. $\rho(v_i, u_{i+1}) = \rho(v_j, u_{j+1})$ for $1 \le i, j \le n-1$
$\Longrightarrow \frac{3i+n}{(2n)^2} = \frac{3j+n}{(2n)^2} \Longrightarrow$ i = j.
2. $\rho(v_i, u_{i-1}) = \rho(v_j, u_{j-1})$ for $2 \le i, j \le n$
$\Longrightarrow \frac{3i-1+n}{(2n)^2} = \frac{3j-1+n}{(2n)^2} \Longrightarrow 3(i-j) = 0 \Longrightarrow$ i = j .
3. $\rho(v_i, u_{i+1}) = \rho(v_j, u_{j-1})$ for $1 \le i \le n-1$ and $2 \le j \le n$
$\Longrightarrow \frac{3i+n}{(2n)^2} = \frac{3j-1+n}{(2n)^2} \Longrightarrow 3(i-j)$ = -1 , not possible.

So m and ρ are injective.

Theorem 3.3

The corona $P_n \odot K_1$ admits fuzzy linear labeling for n ≢ 1(mod 3).

Proof

Consider the corona G = $P_n \odot K_1$ = (V, E), where $|V|$ = 2n,$|E|$= 2n -1.
Let V = ($v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n$).
Define m: V→[0,1] by $m(v_i) = \frac{i}{2n}$, $1 \le i \le n$ and

$m(u_i) = \frac{i+n}{2n}$, $1 \le i \le n$.

Define ρ: E →[0,1] by $\rho(v_i, v_{i+1}) = \frac{3i+1}{(2n)^2}$, $1 \le i \le n-1$

$\rho(v_i, u_i) = \frac{3i+n}{(2n)^2}$, $1 \le i \le n$.

If possible, suppose that m is not injective.

Then there exist some i,j such that

1. $m(v_i)= m(v_j) \Longrightarrow \frac{i}{2n} = \frac{j}{2n} \Longrightarrow i = j$.
2. $m(u_i) = m(u_j) \Longrightarrow \frac{i+n}{2n} = \frac{j+n}{2n} \Longrightarrow i = j$.
3. $m(v_i)= m(u_j) \Longrightarrow \frac{i}{2n} = \frac{j+n}{2n} \Longrightarrow i-j = n$, not possible.

Also if ρ is not injective, then there exist some i, j $\le n$ such that

1. $\rho(v_i, v_{i+1}) = \rho(v_j, v_{j+1})$ for $1 \le i,j \le n-1$
   $\Longrightarrow \frac{3i+1}{(2n)^2} = \frac{3j+1}{(2n)^2} \Longrightarrow i = j$.
2. $\rho(v_i, u_i) = \rho(v_j, u_j)$ for $1 \le i,j \le n$
   $\Longrightarrow \frac{3i+n}{(2n)^2} = \frac{3j+n}{(2n)^2} \Longrightarrow 3(i-j) = 0 \Longrightarrow i = j$.
3. $\rho(v_i, v_{i+1}) = \rho(v_j, u_j)$ for $1 \le i \le n-1$ and $1 \le j \le n$
   $\Longrightarrow \frac{3i+1}{(2n)^2} = \frac{3j+n}{(2n)^2} \Longrightarrow 3(i-j) = n-1 \Longrightarrow n \equiv 1 \pmod 3$

So m and ρ are injective.

Theorem 3.4

The H-graph of a path $P_n$admits fuzzy linear labeling if $5n \not\equiv 0 \pmod 6$ and $n \not\equiv 1 \pmod 6$

Proof

Consider the H-graph G= (V, E) of path graphP_n, where $|V(G)| = 2n$, $|E(G)| = 2n-1$.

Let $V = (v_1,v_2,....,v_n, u_1,u_2,....,u_n)$.

Define m:V→[0,1] by $m(v_i) = \frac{i}{(2n)}$, $1 \le i \le n$ and $m(u_i) = \frac{i+n}{(2n)}$, $1 \le i \le n$

Case (a): When n is Even

Without lossof generality assume that $(v_{\frac{n}{2}}, v_{\frac{n+2}{2}})$is an edge.

Define ρ: E →[0,1] by $\rho(v_i, v_{i+1}) = \frac{3i+1}{(2n)^2}$, $1 \le i \le n-1$.

$\rho(u_i, u_{i+1}) = \frac{3i+3n+1}{(2n)^2}$, $1 \le i \le n-1$.

$\rho(v_{\frac{n}{2}}, u_{\frac{n}{2}+1}) = \frac{5n+2}{2(2n)^2}$.

If possible, suppose that m is not injective.

Then there exist some i,j such that

1. $m(v_i)= m(v_j) \Longrightarrow \frac{i}{2n} = \frac{j}{2n} \Longrightarrow i = j$.
2. $m(u_i) = m(u_j) \Longrightarrow \frac{i+n}{2n} = \frac{j+n}{2n} \Longrightarrow i = j$.
3. $m(v_i)= m(u_j) \Longrightarrow \frac{i}{2n} = \frac{j+n}{2n} \Longrightarrow i-j = n$, not possible.

Also if ρ is not injective, then there exist some i, j $\le n$ such that

1. $\rho(v_i, v_{i+1}) = \rho(v_j, v_{j+1})$ for $1 \le i,j \le n-1$
   $\Longrightarrow \frac{3i+1}{(2n)^2} = \frac{3j+1}{(2n)^2} \Longrightarrow i = j$.
2. $\rho(u_i, u_{i+1}) = \rho(u_j, u_{j+1})$ for $1 \le i,j \le n$-1.
   $\Longrightarrow \frac{3i+3n+1}{(2n)^2} = \frac{3j+3n+1}{(2n)^2} \Longrightarrow 3(i-j) = 0 \Longrightarrow i = j$.
3. $\rho(v_i, v_{i+1}) = \rho(u_j, u_{j+1})$ for $1 \le i,j \le n-1$
   $\Longrightarrow \frac{3i+1}{(2n)^2} = \frac{3j+3n+1}{(2n)^2} \Longrightarrow 3(i-j) = 3n \Longrightarrow n = i-j$, not possible.
4. $\rho(v_i, v_{i+1}) = \rho(v_{\frac{n}{2}}, u_{\frac{n}{2}+1}) \Longrightarrow \frac{3i+1}{(2n)^2} = \frac{5n+2}{2(2n)^2} \Longrightarrow 6i+2 = 5n+2$
   $\Longrightarrow 5n = 6i \Longrightarrow 5n \equiv 0 \pmod 6$.
5. $\rho(u_i, u_{i+1}) = \rho(v_{\frac{n}{2}}, u_{\frac{n}{2}+1}) \Longrightarrow \frac{3i+3n+1}{(2n)^2} = \frac{5n+2}{2(2n)^2} \Longrightarrow$
   $6i+6n+2 = 5n+2$
   $\Longrightarrow n= -6i < 0$, not possible.

So m and ρ are injective.

Case (b): When n is Odd

Without loss of generality assume that $(v_{\frac{n+1}{2}}, v_{\frac{n+1}{2}})$ is an edge.

Define ρ: E →[0,1] by $\rho(v_i, v_{i+1}) = \frac{3i+1}{(2n)^2}$, $1 \le i \le n-1$.

$\rho(u_i, u_{i+1}) = \frac{3i+3n+1}{(2n)^2}$, $1 \le i \le n-1$.

$\rho(v_{\frac{n+1}{2}}, u_{\frac{n+1}{2}}) = \frac{5n+3}{2(2n)^2}$.

If possible, suppose that m is not injective.

Then there exist some i, j such that

1. $m(v_i)= m(v_j) \Longrightarrow \frac{i}{2n} = \frac{j}{2n} \Longrightarrow i = j$.
2. $m(u_i) = m(u_j) \Longrightarrow \frac{i+n}{2n} = \frac{j+n}{2n} \Longrightarrow i = j$.
3. $m(v_i)= m(u_j) \Longrightarrow \frac{i}{2n} = \frac{j+n}{2n} \Longrightarrow i-j = n$, not possible.

Also if ρ is not injective, then there exist some i, j $\le n$ such that

1. $\rho(v_i, v_{i+1}) = \rho(v_j, v_{j+1})$ for $1 \le i,j \le n-1$
   $\Longrightarrow \frac{3i+1}{(2n)^2} = \frac{3j+1}{(2n)^2} \Longrightarrow i = j$.
2. $\rho(u_i, u_{i+1}) = \rho(u_j, u_{j+1})$ for $1 \le i,j \le n$-1.
   $\Longrightarrow \frac{3i+3n+1}{(2n)^2} = \frac{3j+3n+1}{(2n)^2} \Longrightarrow 3(i-j) = 0 \Longrightarrow i = j$.
3. $\rho(v_i, v_{i+1}) = \rho(u_j, u_{j+1})$ for $1 \le i,j \le n-1$
   $\Longrightarrow \frac{3i+1}{(2n)^2} = \frac{3j+3n+1}{(2n)^2} \Longrightarrow 3(i-j) = 3n \Longrightarrow n = i-j$, not possible.
4. $\rho(v_i, v_{i+1}) = \rho(v_{\frac{n+1}{2}}, u_{\frac{n+1}{2}}) \Longrightarrow \frac{3i+1}{(2n)^2} = \frac{5n+3}{2(2n)^2} \Longrightarrow 6i+2 = 5n+3$
   $\Longrightarrow 5n + 1 = 6i \Longrightarrow n \equiv 1 \pmod 6$.
5. $\rho(u_i, u_{i+1}) = \rho(v_{\frac{n+1}{2}}, u_{\frac{n+1}{2}}) \Longrightarrow \frac{3i+3n+1}{(2n)^2} = \frac{5n+3}{2(2n)^2} \Longrightarrow$
   $6i+6n+2 = 5n+3$
   $\Longrightarrow n= -6i+1 < 0$, not possible.

So m and ρ are injective.

IV. CONCLUSION

In this paper we introduced fuzzy linear labeling and some of its properties. We also discuss the admissibility of fuzzy linear labeling in some path related graphs. We show that we can fuzzify the crisp graphs using fuzzy linear labeling.

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