Asian Journal of Engineering and Applied Technology
ISSN: 2249-068X (P) Vol.8 No.2, 2019, pp. 50-56
© The Research Publication, www.trp.org.in
DOI: https://doi.org/10.51983/ajeat-2019.8.2.1138

Study of Properties of Differential Transform Method for Solving the Linear Differential Equation

Nandita Das
Department of Mathematics, Faculty of Science, Islamic University, Kushtia, Bangladesh
E-Mail: nanditadas.math@gmail.com

(Received 20 March 2019; Revised 16 April 2019; Accepted 15 May 2019; Available online 24 May 2019)

***Abstract* - The differential transformation method (DTM) is an alternative procedure for obtaining an analytic Taylor series solution of differential linear and non-linear equations. However, the proofs of the properties of equation have been long ignoredin the DTM literature. In this paper, we present an analytical solution for linear properties of differential equations by using the differential transformation method. This method has been discussed showing the proof of the equation which are presented to show the ability of the method for linear systems of differential equations. Most authors assume the knowledge of these properties, so they do not bother to prove the properties. The properties are therefore proved to serve as a reference for any work that would want to use the properties without proofs. This work argues thatwe can obtain the solution of differential equationthrough these proofs by using the DTM. The result also shows that the technique introduced here is accurate and easy to apply.**

***Keywords:* Differential Transformation Method, DTM, Taylor Series, Linear Properties, Differential Equations**

I. INTRODUCTION

Constructing power-series solutions to differential equations, especially those which do not admit a closed-form solution, has long been an important, and widely-used, solution technique. Traditionally, computing power-series solutions required a fair amount of "boiler-plate" symbolic manipulation, especially in the setup of the power-matching phase. The differential transformation method (DTM) enables the easy construction of a power-series solution by specifying a conversion between the differential equation and a recurrence relation for the power-series coefficients [1, 2]. The differential transformation method (DTM) is an alternative procedure for obtaining an analytic Taylor series solution of differential equations. The main advantage of this method is that it can be applied directly to nonlinear differential equations without requiring linearization and discretization, and therefore, it is not affected by errors associated with discretization.

The concept of DTM was first introduced in the early 1986 by Zhou [3], who solved linear and nonlinear problems in electrical circuits.Differential transformation method (DTM) has been applied to solve linear and non-linear systems of ordinary differential equations [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17] and non-linear system in particular.In this research, we present an analytical solution for linear properties of differential equations by using the differentialtransformation method.

II. MATERIAL AND METHODS

A. Basic Definition: It is well known that if a function *u* is infinitely continuously differentiable, then *u* can be expressed in Taylor series as

$$u(x)=\sum_{k=0}^{\infty}\frac{1}{k!}\frac{d^k u(x_0)}{dx^k}(x-x_0)^k \tag{1}$$

We define the differential transform (DT) of order k, denoted by *U(k)*, by

$$U(k)=\frac{1}{k!}\left[\frac{d^k u(x)}{dx^k}\right]_{x=x_0} \tag{2}$$

In order to solve a given ODE by differential transform method, we make use of the differentialtransform of order *k* given by Equation (2). The differential inverse transform of *U*(*k*) is defined as follows:

$$u(x)=\sum_{k=0}^{\infty}U(k)(x-x_0)^k \tag{3}$$

In real applications, the function *u(x)* is expressed by a finite series and Equation (3) can be truncated, and will be denoted by,

$$u_{(K)}^{DTM}(x)=\sum_{k=0}^{K}U(k)(x-x_0)^k \tag{4}$$

Equation (4) implies that $\sum_{k=K+1}^{\infty}U(k)(x-x_0)^k$ is negligibly small and, in fact, represents the error. The method for calculating this solution is called DTM (*K*).

For convenience, we denote the Differential Transform operator of *k*th order by $\mathfrak{D}_k$ as follows,

$$U(k)=\mathfrak{D}_k[u(x)]=\frac{1}{k!}\left[\frac{d^k u(x)}{dx^k}\right]_{x=x_0}=\frac{1}{k!}D^k u \tag{5}$$

Where D^k represents the kth order derivative with respect to x .

B. Linear Properties of DTM

S. No.	**Original Functions**	**Transformed Functions**
1.	$z(x)=u(x)\pm v(x)$	$Z(k)=U(k)\pm V(k)$
2.	$z(x)=\alpha u(x)$	$Z(k)=\alpha U(k)$
3.	$z(x)=\frac{du(x)}{dx}$	$Z(k)=(k+1)U(k+1)$
4.	$z(x)=\frac{d^2u(x)}{dx^2}$	$Z(k)=(k+1)(k+2)U(k+2)$
5.	$z(x)=\frac{d^mu(x)}{dx^m}$	$Z(k)=(k+1)(k+2).........(k+m)U(k+m)$ $=\frac{(k+m)!}{k!}U(k+m)$
6.	$z(x)=u(x)v(x)$	$Z(k)=\sum_{m=0}^{k}U(m)V(k-m)$
7.	$z(x)=u_1(x)u_2(x)u_3(x)...u_n(x)$	$\sum_{k_{n-1}=0}^{k}\sum_{k_{n-2}=0}^{k_{n-1}}...\sum_{k_2=0}^{k_3}\sum_{k_1=0}^{k_2}U_1(k_1)U_2(k_2-k_1)U_3(k_3-k_2)...U_n(k-k_{n-1})$
8.	$z(x)=x^m$	$Z(k)=\delta(k-m)$, where $\delta(k-m)=\begin{cases}1 & if\ k=m\\ 0 & if\ k\neq m\end{cases}$
9.	$z(x)=\alpha x^m$	$Z(k)=\alpha\delta(k-m)$
10.	$z(x)=(1+x)^m$	$Z(k)=\frac{m(m-1)\cdots(m-k+1)}{k!}$
11.	$z(x)=\int_0^x u(t)dt$	$Z(k)=\frac{U(k-1)}{k}$
12.	$z(x)=e^x$	$Z(k)=\frac{1}{k!}$
13.	$z(x)=e^{\lambda x}$	$Z(k)=\frac{\lambda^k}{k!}$
14.	$z(x)=\sin(\omega x+\alpha)$	$Z(k)=\frac{\omega^k}{k!}\sin(\frac{k\pi}{2}+\alpha)$
15.	$z(x)=\cos(\omega x+\alpha)$	$Z(k)=\frac{\omega^k}{k!}\cos(\frac{k\pi}{2}+\alpha)$

III. FINDINGS OF THE STUDY

A. Proof of Linear Properties

1. $z(x)=u(x)\pm v(x)$

Proof:

$$Z(k)=\mathfrak{D}_k\left[z(x)\right]=\mathfrak{D}_k\left[u(x)\pm v(x)\right]=\frac{1}{k!}\left[\frac{d^k}{dx^k}\left\{u(x)\pm v(x)\right\}\right]_{x=0}$$

$$=\frac{1}{k!}\left[\frac{d^k}{dx^k}u(x)\right]_{x=0}+\frac{1}{k!}\left[\frac{d^k}{dx^k}v(x)\right]_{x=0}=U(k)\pm V(k)$$

$$\therefore Z(k)=U(k)\pm V(k)$$

2. $z(x) = \alpha u(x)$

Proof:

$$Z(k) = \mathfrak{D}_k\left[z(x)\right] = \mathfrak{D}_k\left[\alpha u(x)\right] = \frac{1}{k!}\left[\frac{d^k}{dx^k}\left\{\alpha u(x)\right\}\right]_{x=0}$$

$$= \alpha\frac{1}{k!}\left[\frac{d^k}{dx^k}u(x)\right]_{x=0} = \alpha U(k)$$

$$\therefore Z(k) = \alpha U(k)$$

3. $z(x) = \dfrac{du(x)}{dx}$

Proof:

$$Z(k) = \mathfrak{D}_k\left[z(x)\right] = \mathfrak{D}_k\left[\frac{du(x)}{dx}\right] = \frac{1}{k!}\left[\frac{d^k}{dx^k}\left\{\frac{d}{dx}u(x)\right\}\right]_{x=0} = \frac{1}{k!}\left[D^k\left(Du\right)\right]_{x=0}$$

$$= \frac{1}{k!}\left[D^{k+1}\left(u\right)\right]_{x=0} = \frac{1}{k!}\left[\frac{d^{k+1}}{dx^{k+1}}u(x)\right]_{x=0} = \frac{1}{k!}\frac{\left(k+1\right)!}{\left(k+1\right)!}\left[\frac{d^{k+1}}{dx^{k+1}}u(x)\right]_{x=0}$$

$$= \frac{\left(k+1\right)k!}{k!}\frac{1}{\left(k+1\right)!}\left[\frac{d^{k+1}}{dx^{k+1}}u(x)\right]_{x=0} = \left(k+1\right)\frac{1}{\left(k+1\right)!}\left[\frac{d^{k+1}}{dx^{k+1}}u(x)\right]_{x=0}$$

$$= \left(k+1\right)\mathfrak{D}_{k+1}\left[u(x)\right]_{x=0} = \left(k+1\right)U\left(k+1\right)$$

$$\therefore Z(k) = (k+1)U(k+1)$$

4. $z\left(x\right) = \dfrac{d^2u\left(x\right)}{dx^2}$

Proof: $Z(k) = \mathfrak{D}_k\left[z(x)\right] = \mathfrak{D}_k\left[\dfrac{d^2u(x)}{dx^2}\right] = \dfrac{1}{k!}\left[\dfrac{d^k}{dx^k}\left\{\dfrac{d^2}{dx^2}u(x)\right\}\right]_{x=0}$

$$= \frac{1}{k!}\left[D^k\left(D^2u\right)\right]_{x=0} = \frac{1}{k!}\left[D^{k+2}\left(u\right)\right]_{x=0} = \frac{1}{k!}\left[\frac{d^{k+2}}{dx^{k+2}}u(x)\right]_{x=0}$$

$$= \frac{1}{k!}\frac{(k+2)!}{(k+2)!}\left[\frac{d^{k+2}}{dx^{k+2}}u(x)\right]_{x=0} = \frac{\left(k+2\right)\left(k+1\right)k!}{k!}\frac{1}{\left(k+2\right)!}\left[\frac{d^{k+2}}{dx^{k+2}}u(x)\right]_{x=0}$$

$$= \left(k+2\right)\left(k+1\right)\frac{1}{\left(k+2\right)!}\left[\frac{d^{k+2}}{dx^{k+2}}u(x)\right]_{x=0} = \left(k+2\right)\left(k+1\right)\mathfrak{D}_{k+2}\left[u(x)\right]_{x=0}$$

$$= \left(k+2\right)\left(k+1\right)U\left(k+2\right)$$

$$\therefore Z(k) = \left(k+1\right)(k+2)U(k+2)$$

5. $z\left(x\right)=\dfrac{d^{m}u\left(x\right)}{dx^{m}}$

Proof: $Z(k)=\mathcal{D}_k\left[z(x)\right]=\mathcal{D}_k\left[\dfrac{d^{m}u(x)}{dx^{m}}\right]=\dfrac{1}{k!}\left[\dfrac{d^{k}}{dx^{k}}\left\{\dfrac{d^{m}}{dx^{m}}u(x)\right\}\right]_{x=0}=\dfrac{1}{k!}\left[D^{k}\left(D^{m}u\right)\right]_{x=0}$

$$=\frac{1}{k!}\left[D^{k+m}\left(u\right)\right]_{x=0}=\frac{1}{k!}\left[\frac{d^{k+m}}{dx^{k+m}}u(x)\right]_{x=0}=\frac{1}{k!}\frac{\left(k+m\right)!}{\left(k+m\right)!}\left[\frac{d^{k+m}}{dx^{k+m}}u(x)\right]_{x=0}$$

$$=\frac{\left(k+m\right)\left(k+m-1\right)\cdots\left(k+m-m\right)\left(k-1\right)\cdots2.1}{k!}\frac{1}{\left(k+m\right)!}\left[\frac{d^{k+m}}{dx^{k+m}}u(x)\right]_{x=0}$$

$$=\frac{\left(k+m\right)\left(k+m-1\right)\cdots\left(k+m-m+1\right)k!}{k!}\mathcal{D}_{k+m}\left[u(x)\right]_{x=0}$$

$$=\frac{\left(k+1\right)\left(k+2\right)\cdots\left(k+m\right)k!}{k!}U\left(k+m\right)$$

$$=\left(k+1\right)\left(k+2\right)\cdots\left(k+m\right)U\left(k+m\right)$$

$$\therefore Z(k)=\left(k+1\right)(k+2)\cdots\left(k+m\right)U(k+m)$$

6. $z\left(x\right)=u\left(x\right)v\left(x\right)$

Proof:

$$\therefore Z(k)=\mathcal{D}_k\left[z(x)\right]=\mathcal{D}_k\left[u\left(x\right)v\left(x\right)\right]=\frac{1}{k!}\left[\frac{d^{k}}{dx^{k}}\left\{u\left(x\right)v\left(x\right)\right\}\right]_{x=0}=\frac{1}{k!}\left[D^{k}\left(uv\right)\right]_{x=0}$$

$$=\frac{1}{k!}\left\{\left(D^{k}u\right).v+k\left(D^{k-1}u\right).Dv+\frac{k\left(k-1\right)}{2!}\left(D^{k-2}u\right).D^{2}v+...+k\left(Du\right).\left(D^{k-1}v\right)+u.\left(D^{k}v\right)\right\}$$

[by Leibnitz's Theorem]

$$=\left(\frac{1}{k!}D^{k}u\right)\frac{1}{0!}D^{0}v+\left(\frac{1}{\left(k-1\right)!}D^{k-1}u\right)\frac{1}{1!}D^{1}v+\left(\frac{1}{\left(k-2\right)!}D^{k-2}u\right)\frac{1}{2!}D^{2}v+...$$

$$+\left(\frac{1}{1!}D^{1}u\right)\frac{1}{\left(k-1\right)!}D^{k-1}v+\left(\frac{1}{0!}D^{0}u\right)\frac{1}{k!}D^{k}v$$

$$=\sum_{l=0}^{k}\left(\frac{1}{\left(k-l\right)!}D^{k-l}u\right)\left(\frac{1}{l!}D^{l}v\right)=\sum_{l=0}^{k}U\left(k-l\right)V\left(l\right)$$

$$=\sum_{m=k}^{m=0}U\left(m\right)V\left(k-m\right)=\sum_{m=0}^{k}U\left(m\right)V\left(k-m\right)$$

$$\therefore Z(k)=\sum_{m=0}^{k}U(m)V(k-m)$$

7. $z\left(x\right)=u_{1}\left(x\right)u_{2}\left(x\right)u_{3}\left(x\right)\cdots u_{n}\left(x\right)$

Proof:
Case I: $n=2$ i.e., product of two functions:

$$z(x)=u_1(x)u_2(x)$$

$$\mathcal{D}_k\left[z(x)\right]=\sum_{m=0}^{k}U_1(m)U_2(k-m)$$

When $m\to k_1$,

$$\mathcal{D}_k\left[z(x)\right]=\sum_{k_1=0}^{k}U_1(k_1)U_2(k-k_1)$$

Case II: $n=3$ i.e., product of three functions:

$$z(x)=u_1(x)u_2(x)u_3(x)$$

$$\mathcal{D}_k\left[z(x)\right]=\sum_{l=0}^{k}U(l)U_3(k-l),$$

where $U(l)=\mathcal{D}_l\left[u_1(x)u_2(x)\right]$ and

$U_3(k-l)=\mathcal{D}_{k-l}\left[u_3(x)\right]$.

$$\therefore\mathcal{D}_k\left[z(x)\right]=\sum_{l=0}^{k}\sum_{m=0}^{l}U_1(m)U_2(l-m)U_3(k-l),$$

by using Case I.

When $m\to k_1, l\to k_2$,

$$\therefore\mathcal{D}_k\left[z(x)\right]=\sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2}U_1(k_1)U_2(k_2-k_1)U_3(k-k_2)$$

Case III: $n=4$ i.e., product of four functions:

$$z(x)=u_1(x)u_2(x)u_3(x)u_4(x)$$

$$\mathcal{D}_k\left[z(x)\right]=\sum_{p=0}^{k}U(p)U_4(k-p),$$

where $U(p)=\mathcal{D}_p\left[u_1(x)u_2(x)u_3(x)\right]$ and

$U_3(k-p)=\mathcal{D}_{k-p}\left[u_3(x)\right]$.

$$\therefore\mathcal{D}_k\left[z(x)\right]=\sum_{p=0}^{k}\sum_{k_2=0}^{p}\sum_{k_1=0}^{k_2}U_1(k_1)U_2(k_2-k_1)U_3(p-k_2)U_4(k-p)$$

When $p\to k_3$,

$$\mathcal{D}_k\left[z(x)\right]=\sum_{k_3=0}^{k}\sum_{k_2=0}^{k_3}\sum_{k_1=0}^{k_2}U_1(k_1)U_2(k_2-k_1)U_3(k_3-k_2)U_4(k-k_3)$$

General Case: Proceeding as above, for $z(x)=u_1(x)u_2(x)u_3(x)\cdots u_n(x)$ we obtain

$$Z(k)=\mathcal{D}_k\left[z(x)\right]=\sum_{k_{n-1}=0}^{k}\sum_{k_{n-2}=0}^{k_{n-1}}\cdots\sum_{k_2=0}^{k_3}\sum_{k_1=0}^{k_2}U_1(k_1)U_2(k_2-k_1)U_3(k_3-k_2)\cdots U_n(k-k_{n-1})$$

8. $z(x)=x^m$

Proof: We have,

$$D^k\left(x^m\right)=\frac{d^k}{dx^k}\left(x^m\right)=m\frac{d^{k-1}}{dx^{k-1}}\left(x^{m-1}\right)$$

$$=m(m-1)\frac{d^{k-2}}{dx^{k-2}}\left(x^{m-2}\right)$$

$$=m(m-1)\cdots\left(m-\overline{k-1}\right)\frac{d^{k-m}}{dx^{k-m}}\left(x^{m-k}\right)$$

$$=\begin{cases}m(m-1)\cdots 2.1=m! & if\ \ k=m\\ 0 & if\ \ k\neq m\end{cases}$$

$$\therefore Z(k)=\mathcal{D}_k\left[z(x)\right]=\frac{1}{k!}\left[D^k\left(x^m\right)\right]_{x=0}$$

$$=\frac{1}{k!}\begin{cases}m(m-1)\cdots 2.1=m! & if\ \ k=m\\ 0 & if\ \ k\neq m\end{cases}$$

$$=\begin{cases}1 & if\ \ k=m\\ 0 & if\ \ k\neq m\end{cases}$$

$$=\delta(k-m)$$

$$\therefore Z(k)=\delta(k-m)$$

9. $z(x)=\alpha x^m$

Proof: We have,

$$D^k\left(x^m\right)=\frac{d^k}{dx^k}\left(x^m\right)=m\frac{d^{k-1}}{dx^{k-1}}\left(x^{m-1}\right)$$

$$=m(m-1)\frac{d^{k-2}}{dx^{k-2}}\left(x^{m-2}\right)$$

$$=m(m-1)\cdots\left(m-\overline{k-1}\right)\frac{d^{k-m}}{dx^{k-m}}\left(x^{m-k}\right)$$

$$=\begin{cases}m(m-1)\cdots 2.1=m! & if\ \ k=m\\ 0 & if\ \ k\neq m\end{cases}$$

$$\therefore Z(k)=\mathcal{D}_k\left[z(x)\right]=\frac{1}{k!}\left[D^k\left(\alpha x^m\right)\right]$$

$$=\alpha\frac{1}{k!}\begin{cases}m(m-1)\cdots 2.1=m! & if\ \ k=m\\ 0 & if\ \ k\neq m\end{cases}$$

$$= \alpha \begin{cases} 1 & if \ k = m \\ 0 & if \ k \neq m \end{cases}$$

$$= \alpha\, \delta(k-m)$$

$$\therefore Z(k) = \alpha\ \delta(k-m)$$

10. $z(x) = (1+x)^m$

Proof: $Z(k) = \mathfrak{D}_k\left[z(x)\right] = \mathfrak{D}_k\left[(1+m)^m\right]$

$$= \frac{1}{k!}\left[\frac{d^k}{dx^k}(1+x)^m\right]_{x=0}$$

$$= \frac{1}{k!}\left[\frac{d^{k-1}}{dx^{k-1}}\left\{m(1+x)^{m-1}\right\}\right]_{x=0}$$

$$= \frac{1}{k!}\left[\frac{d^{k-2}}{dx^{k-2}}\left\{m(m-1)(1+x)^{m-2}\right\}\right]_{x=0}$$

$$= \frac{1}{k!}\left[\frac{d^{k-r}}{dx^{k-r}}\left\{m(m-1)\cdots(m-r+1)(1+x)^{m-r}\right\}\right]_{x=0}$$

$$= \frac{1}{k!}\left[m(m-1)\cdots(m-k+1)(1+x)^{m-k}\right]_{x=0}$$

$$= \frac{1}{k!}m(m-1)(m-2)\cdots(m-k+1).1$$

$$= \frac{m(m-1)(m-2)\cdots(m-k+1)}{k!}$$

$$\therefore Z(k) = \frac{m(m-1)\cdots(m-k+1)}{k!}$$

11. $z(x) = \int\limits_0^x u(t)\,dt$

Proof: $Z(k) = \mathfrak{D}_k\left[z(x)\right] = \mathfrak{D}_k\left[\int\limits_0^x u(t)\,dt\right]$

$$= \frac{1}{k!}\left[\frac{d^k}{dx^k}\left\{\int\limits_0^x u(t)\,dt\right\}\right]_{x=0}$$

$$= \frac{1}{k!}\left[\frac{d^{k-1}}{dx^{k-1}}\left\{u(x)\right\}\right]_{x=0}$$

$$= \frac{1}{k!}\frac{(k-1)!}{(k-1)!}\left[\frac{d^{k-1}}{dx^{k-1}}u(x)\right]_{x=0}$$

$$= \frac{(k-1)!}{k!}\frac{1}{(k-1)!}\left[\frac{d^{k-1}}{dx^{k-1}}u(x)\right]_{x=0}$$

$$= \frac{(k-1)!}{k(k-1)!}\mathfrak{D}_{k-1}\left[u(x)\right]_{x=0} = \frac{1}{k}U(k-1)$$

$$= \frac{U(k-1)}{k}$$

$$\therefore Z(k) = \frac{U(k-1)}{k}$$

12. $z(x) = e^x$

Proof: $Z(k) = \mathfrak{D}_k\left[z(x)\right] = \frac{1}{k!}\left[D^k\left(e^x\right)\right]_{x=0}$

$$= \frac{1}{k!}\left[\frac{d^k}{dx^k}e^x\right]_{x=0} = \frac{1}{k!}\left[e^x\right]_{x=0}$$

$$= \frac{1}{k!}e^0 = \frac{1}{k!}.1 = \frac{1}{k!}$$

$$\therefore Z(k) = \frac{1}{k!}$$

13. $z(x) = e^{\lambda x}$

Proof: $Z(k) = \mathfrak{D}_k\left[z(x)\right] = \frac{1}{k!}\left[D^k\left(e^{\lambda x}\right)\right]_{x=0}$

$$= \frac{1}{k!}\left[\frac{d^k}{dx^k}e^{\lambda x}\right]_{x=0} = \frac{1}{k!}\left[\lambda^k e^{\lambda x}\right]_{x=0}$$

$$= \frac{1}{k!}\lambda^k\left[e^{\lambda x}\right]_{x=0} = \frac{\lambda^k}{k!}e^0 = \frac{\lambda^k}{k!}.1 = \frac{\lambda^k}{k!}$$

$$\therefore Z(k) = \frac{\lambda^k}{k!}$$

14. $z(x) = \sin(\omega x + \alpha)$

Proof: Let $z(x) = y = \sin(\omega x + \alpha)$

$$\therefore y_1 = \omega\cos(\omega x + \alpha) = \omega\sin\left(\frac{\pi}{2} + \omega x + \alpha\right)$$

$$\therefore y_2 = \omega^2\cos\left(\frac{\pi}{2} + \omega x + \alpha\right) = \omega^2\sin\left(\frac{2\pi}{2} + \omega x + \alpha\right)$$

.

.

.

$$\therefore y_r = \omega^r\sin\left(\frac{r\pi}{2} + \omega x + \alpha\right)$$

Hence $Z(k) = \mathfrak{D}_k\left[z(x)\right] = \frac{1}{k!}\left[\frac{d^k}{dx^k}\left\{\sin(\omega x + \alpha)\right\}\right]_{x=0}$

$$=\frac{1}{k!}\omega^{k}\left[\sin\left(\frac{k\pi}{2}+\omega x+\alpha\right)\right]_{x=0}=\frac{1}{k!}\omega^{k}\sin\left(\frac{k\pi}{2}+\alpha\right)$$

$$\therefore Z(k)=\frac{\omega^{k}}{k!}\sin(\frac{k\pi}{2}+\alpha)$$

15. $z\left(x\right)=\cos\left(\omega x+\alpha\right)$

Proof: Let, $z\left(x\right)=y=\cos\left(\omega x+\alpha\right)$

$$\therefore y_{1}=-\omega\sin\left(\omega x+\alpha\right)=\omega\cos\left(\frac{\pi}{2}+\omega x+\alpha\right)$$

$$\therefore y_{2}=-\omega^{2}\sin\left(\frac{\pi}{2}+\omega x+\alpha\right)=\omega^{2}\cos\left(\frac{2\pi}{2}+\omega x+\alpha\right)$$

$$\vdots$$

$$\therefore y_{r}=\omega^{r}\cos\left(\frac{r\pi}{2}+\omega x+\alpha\right)$$

Hence, $Z(k)=\mathfrak{D}_{k}\left[z(x)\right]=\frac{1}{k!}\left[\frac{d^{k}}{dx^{k}}\left\{\cos\left(\omega x+\alpha\right)\right\}\right]_{x=0}$

$$=\frac{1}{k!}\omega^{k}\left[\cos\left(\frac{k\pi}{2}+\omega x+\alpha\right)\right]_{x=0}=\frac{1}{k!}\omega^{k}\cos\left(\frac{k\pi}{2}+\alpha\right)$$

$$\therefore Z(k)=\frac{\omega^{k}}{k!}\cos(\frac{k\pi}{2}+\alpha)$$

IV. CONCLUSION

This study has tried to prove the linear properties whose proofs have been long ignored in the DTM literature. Most authors assume the knowledge of these properties, so they do not bother to prove the properties. The properties are therefore proved to serve as a reference for any work that would want to use the properties without proofs. This work argues that we can obtain the solution of differential equation through these proofs by using the DTM.

REFERENCES

[1] C. Chen and S. Ho, "Application of differential transformation to eigenvalue problems", *Applied Mathematics and Computation*, Vol. 79, No. 2-3, pp. 173-188, 1996. [Online] Available at: 10.1016/0096-3003(95)00253-7.

[2] M. Hatami, D. Ganji and M. Sheikholeslami, "Differential transformation method for mechanical engineering problems". *Elsevier, 2003*.

[3] J. Zhou, "Differential Transformation and Its Applications for Electrical Circuits" (in Chinese), *Huazhong University Press*, 1986.

[4] I. Abdel-Halim Hassan, "Comparison differential transformation technique with Adomian decomposition method for linear and nonlinear initial value problems", *Chaos, Solitons & Fractals*, Vol. 36, No. 1, pp. 53-65, 2008. [Online] Available at: 10.1016/j.chaos.2006.06.040.

[5] J. Biazar and H. Ghazvini, "He's variational iteration method for solving linear and non-linear systems of ordinary differential equations", *Applied Mathematics and Computation*, Vol. 191, No. 1, pp. 287-297, 2007. [Online] Available at: 10.1016/j.amc.2007.02.153.

[6] C. Kuang Chen and S. Huei Ho, "Solving partial differential equations by two-dimensional differential transform method", *Applied Mathematics and Computation*, Vol. 106, No. 2-3, pp. 171-179, 1999. [Online] Available at: 10.1016/s0096-3003(98)10115-7.

[7] F. Ayaz, "Solutions of the system of differential equations by differential transform method", *Applied Mathematics and Computation*, Vol. 147, No. 2, pp. 547-567, 2004. [Online] Available at: 10.1016/s0096-3003(02)00794-4.

[8] Z. Odibat, "Differential transform method for solving Volterra integral equation with separable kernels", *Mathematical and Computer Modelling*, Vol. 48, No. 7-8, pp. 1144-1149, 2008. [Online] Available at: 10.1016/j.mcm.2007.12.022.

[9] F. Kangalgil and F. Ayaz, "Solitary wave solutions for the KdV and mKdV equations by differential transform method", *Chaos, Solitons & Fractals*, Vol. 41, No. 1, pp. 464-472, 2009. [Online] Available at: 10.1016/j.chaos.2008.02.009.

[10] M. Jang, C. Chen and Y. Liu, "Two-dimensional differential transform for partial differential equations", *Applied Mathematics and Computation*, Vol. 121, No. 2-3, pp. 261-270, 2001. [Online] Available at: 10.1016/s0096-3003(99)00293-3.

[11] I. Abdel-Halim Hassan, "Different applications for the differential transformation in the differential equations", *Applied Mathematics and Computation*, Vol. 129, No. 2-3, pp. 183-201, 2002. [Online] Available at: 10.1016/s0096-3003(01)00037-6.

[12] F. Ayaz, "On the two-dimensional differential transform method", *Applied Mathematics and Computation*, Vol. 143, No. 2-3, pp. 361-374, 2003. [Online] Available at: 10.1016/s0096-3003(02) 00368-5.

[13] A. Arikoglu and I. Ozkol, "Solution of boundary value problems for integro-differential equations by using differential transform method", *Applied Mathematics and Computation*, Vol. 168, No. 2, pp. 1145-1158, 2005. [Online] Available at: 10.1016/j.amc.2004.10. 009.

[14] A. Kurnaz, G. Oturanç and M. Kiris, "n-Dimensional differential transformation method for solving PDEs", *International Journal of Computer Mathematics*, Vol. 82, No. 3, pp. 369-380, 2005. [Online] Available at: 10.1080/0020716042000301725.

[15] A. Ravi Kanth and K. Aruna, "Differential transform method for solving linear and non-linear systems of partial differential equations", *Physics Letters A*, Vol. 372, No. 46, pp. 6896-6898, 2008. [Online] Available at: 10.1016/j.physleta.2008.10.008.

[16] A. Ravi Kanth and K. Aruna, "Differential transform method for solving the linear and nonlinear Klein–Gordon equation", *Computer Physics Communications*, Vol. 180, No. 5, pp. 708-711, 2009. [Online] Available: 10.1016/j.cpc.2008.11.012.

[17] F. Mirzaee, "Differential transform method for solving linear and nonlinear systems of ordinary differential equations", *Applied Mathematical Sciences*, Vol. 05, No. 70, pp. 3465 - 3472, 2011.