Perturbation Analysis of Rivlin-Ericksen Fluid on Heat Transfer in the Presence of Heat Absorption

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(Received 23 June 2019; Revised 15 July 2019; Accepted 10 August 2019; Available online 18 August 2019)

Abstract - The problem of visco-elastic Rivlin-Ericksen fluid flow past a semi- infinite vertical plate embedded in a porous medium with variable temperature and suction in the presence of a uniform transverse magnetic field and thermal buoyancy effect is considered. The plate is assumed to move with a constant velocity in the direction of fluid flow while the free stream velocity is assumed to follow the exponentially increasing small perturbation law. Time-dependent wall suction is assumed to occur at the permeable surface. The dimensionless governing equations for this investigation are solved analytically using two-term harmonic and nonharmonic functions. Numerical evaluation of the analytical results is performed and some graphical results such as viscoelastic parameter Rm, heat absorption parameter Q, Grashof number Gr, Prandtl number Pr, time t, suction velocity parameter A, moving velocity parameter Up and an exponential parameter ε , for the velocity and temperature profiles within the boundary layer are presented. Skin-friction coefficient, Nusselt numbers are also discussed with the help of the tables.

Keywords: Variable Temperature, Vertical Plate, Suction, Heat Absorption, Rivlin-Ericksen Flow

I. INTRODUCTION

A few modern applications include the progression of non-Newtonian liquids, and in this manner the stream conduct of such liquids finds an incredible pertinence. Moltenmetal's, plastic, pulps, emulsions; slurries and crude materials in liquid state are a few guides to make reference to. Non-Newtonian stream likewise finds functional applications in bio-designing, wherein blood flows brutal/creature vein is clarified by a fitting Visco-versatile liquid model of little flexibility. The investigation of a visco-versatile pulsatile stream helps in understanding the component of dialysis of blood through a counterfeit kidney. A significant class of two dimensional time ward stream issue managing the reaction of limit layer to outside precarious changes of the free stream speed about a mean worth pulled in the consideration of numerous specialists. Other than that convective course through permeable medium has an application in geothermal vitality recuperation, warm vitality stockpiling, oil extraction, and move through separating gadgets. Presently, magnetohydrodynamics is particularly pulling in the consideration of the numerous creators because of its applications in geophysics and building. MHD stream with warmth and mass exchange has been a subject of enthusiasm of numerous specialists in light of its different applications in science and innovation. Such wonders are seen in lightness initiated movements in the climate, in water bodies, semi strong bodies, for example, earth, and so on. The progression of viscous incompressible fluid past an indiscreetly started infinite horizontal plate in its own plane was first examined by Stokes [1]. Convective heat transfer Non-Newtonian MHD flow permeable plate inspected by the authors [2-5]. Magnificent assessments have the made on visco-elastic impact by the author [6].

An examination of viscous dissipation impacts on free convection heat and mass transfer of MHD non-Newtonian fluid flow through a permeable medium given by Nabil et al., [7]. Unsteady flow of a non-Newtonian fluid over a rotating disk with heat transfer examined by Attia [8]. MHD Rivlin-Ericksen flow through porous medium in slip stream system examined by Ramana Murthy et al., [9]. Unsteady MHD free convective heat and mass transfer flow vertical permeable plate discussed by Rao et al., [10]. Unsteady MHD free convection flow of a Kuvshinski fluid past a vertical permeable plate assumed by Reddy et al., [11]. Unsteady MHD free convection boundary layer flow of radiation on Kuvshinski fluid through porous medium was reported by Vidyasagar et al., [12]. Propelled by these examinations referenced over, the motivation behind the present work is extended the investigation of Kim [13], by considering a notable non-Newtonian fluid namely Rivlin-Ericksen fluid for the instance of a semi-infinite moving porous plate in a permeable medium. We likewise consider the free stream to comprise of a mean velocity and temperature over which are superimposed an exponentially shifting with time. The obtained outcomes here are in great concurrence with the results of Kim without non-Newtonian fluid. The problem of visco-elastic Rivlin-Ericksen fluid past a semi- infinite vertical plate embedded in a permeable medium with variable temperature and suction in the presence of a uniform transverse magnetic field and thermal buoyancy effect is assumed. The plate is considered to move with a steady velocity in the direction of fluid flow while the free stream velocity is expected to follow the exponentially expanding small perturbation law. Timedependent wall suction is accepted to happen at the

penetrable surface. The dimensionless governing conditions for this examination are unraveled diagnostically utilizingtwo-term harmonic and non-harmonic functions. Numerical assessment of the scientific results is performed and some graphical outcomes, such as visco-elastic parameter Rm, heat absorption parameter Q, Grashof number Gr, Prandtl number Pr, time t, suction velocity parameter A, moving velocity parameter Up and an exponential parameter ε , for the velocity and temperature profiles within the boundary layer are exhibited. Skinfriction coefficient and Nusselt numbers are additionally examined with the assistance of the tables.

II. FORMULATION OF THE PROBLEM

We consider transient 2-dimensional flow of a laminar, incompressible, viscous, electrically conducting and heat absorbing Rivlin–Ericksen fluid past a semi-infinite permeable vertical moving plate.

 $\frac{\partial \tilde{u}}{\partial \tilde{t}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial \tilde{x}} - k_{v}\left[\frac{\partial^{3}\tilde{u}}{\partial \tilde{t}\partial \tilde{y}^{2}} + \tilde{v}\frac{\partial^{3}\tilde{u}}{\partial \tilde{y}^{3}}\right] - \frac{\sigma B_{0}^{2}}{\rho}\tilde{u} - v\frac{\tilde{u}}{\tilde{k}} + g\beta_{T}\left[\tilde{T} - \tilde{T}_{\infty}\right] + v\frac{\partial^{2}\tilde{u}}{\partial \tilde{y}^{2}}$ (2)

$$\frac{\partial \tilde{T}}{\partial \tilde{t}} + \tilde{v} \frac{\partial \tilde{T}}{\partial \tilde{y}} = \alpha_d \frac{\partial^2 \tilde{T}}{\partial \tilde{y}^2} - \frac{\tilde{Q}}{\rho C p} \Big[\tilde{T} - \tilde{T}_{\infty} \Big]$$
(3)

The boundary layer of equation (2) gives

$$\frac{dU_{\infty}}{d\tilde{t}} + \frac{v}{\tilde{k}}\tilde{U}_{\infty} + \frac{\sigma}{\rho}\tilde{U}_{\infty}B_0^2 = -\frac{1}{\rho}\frac{\partial\tilde{p}}{\partial\tilde{x}}$$
(4)

Corresponding boundary conditions are

$$\tilde{u} = \tilde{u}_p, T = T_w + \varepsilon e^{\tilde{n}\tilde{t}} \left[\tilde{T}_w - \tilde{T}_\infty \right] \quad at \ \tilde{y} = 0 \tag{5}$$

$$\tilde{u} \to \tilde{u}_{\infty} = U_0 \Big[1 + \varepsilon e^{\tilde{n}\tilde{t}} \Big], T \to T_{\infty} \quad as \; \tilde{y} \to \infty \tag{6}$$

The suction velocity at the plate surface of equation (1) is a function of time only so it assumes

$$\tilde{\nu} = -V_0 \left(1 + \varepsilon A e^{\tilde{n} \tilde{t}} \right) \tag{7}$$

Where \tilde{x} - the dimensional distances along the plate, \tilde{y} - the dimensional distance perpendicular to the plate, \tilde{t} - dimensional time, \tilde{u} - the component of dimensional velocity along \tilde{x} direction, \tilde{v} - the component of dimensional velocity along \tilde{y} direction, ρ – the fluid density, v – the kinematic viscosity, Cp – the specific heat at constant pressure, σ – the fluid electrical conductivity, B_0 - magnetic induction, \tilde{k} - the permeability of the porous medium, T – the dimensional temperature, Q_0 – the dimensional heat absorption coefficient, α_d -the thermal diffusivity, g – gravitational acceleration, β_T - the coefficient of volumetric expansion, k_v - the kinematic visco-elasticity, A – a real positive constant, $\varepsilon \& \varepsilon A$ – less than unity values, V_0 – a scale of suction velocity which has non-zero positive constant.

Here we considered the following:

- 1. There is no applied voltage that implies the absence of an electrical field.
- 2. The magnetic field and Reynolds number are very small.
- 3. The Hall effects are negligible.
- 4. Viscous dissipation is neglected.
- 5. Plate moves with a constant velocity in the direction of fluid flow.
- 6. The temperature at the wall as well as the suction velocity is exponentially varying with time.

The governing equations can be written in Cartesian frame of reference as:

$$\frac{\partial \tilde{v}}{\partial \tilde{y}} = 0 \tag{1}$$

$$u = \frac{\tilde{u}}{U_0}, v = \frac{\tilde{v}}{V_0}, \eta = \frac{V_0 \tilde{y}}{v}, U_{\infty} = \frac{\tilde{U}_{\infty}}{U_0}, R_m = \frac{V_0^2 k_v}{v^2}, k = \frac{\tilde{k} V_0^2}{v^2}, t = \frac{\tilde{t} V_0^2}{v}, \theta = \frac{\tilde{T} - \tilde{T}_{\infty}}{\tilde{T}_w - \tilde{T}_{\infty}}, n = \frac{\tilde{n} v}{V_0^2}, \Pr = \frac{v}{\alpha_d} = \frac{v \rho C_p}{k},$$
(8)
$$M = \frac{\sigma v B_0^2}{\rho V_0^2}, Gr = \frac{v \beta_T g \left[\tilde{T}_w - \tilde{T}_{\infty} \right]}{U_0 V_0^2}, Q = \frac{v Q_0}{V_0^2 \rho C_p}$$

Substituting using equation (8) along with (4) to (7) in equations (1) and (2), we can obtain dimensionless forms in the following manner:

$$\frac{\partial u}{\partial t} - \left[1 + \varepsilon A e^{nt}\right] \frac{\partial u}{\partial \eta} = \frac{dU_{\infty}}{dt} - Rm \left[\frac{\partial^3 u}{\partial t \partial \eta^2} - \left[1 + \varepsilon A e^{nt}\right] \frac{\partial^3 u}{\partial \eta^3}\right]_{(9)} + N_1 \left[U_{\infty} - u\right] + Gr\theta + \frac{\partial^2 u}{\partial \eta^2}$$

$$\Pr\left(\frac{\partial \theta}{\partial t} - \left[1 + \varepsilon A e^{nt}\right] \frac{\partial \theta}{\partial \eta}\right] = \frac{\partial^2 \theta}{\partial \eta^2} - \Pr Q\theta \qquad (10)$$

The corresponding dimensionless boundary conditions are

$$u = U_p, \theta = 1 + \varepsilon e^{nt} \qquad at \quad \eta = 0 \tag{11}$$

$$u \to U_{\infty} = U_0 \Big[1 + \varepsilon e^{nt} \Big] \qquad \text{as} \quad \eta \to \infty \tag{12}$$

Where

$$N_1 = M + \frac{1}{k};$$

Also Gr – the solutal Grashof number, Pr – Prandtl number, Q – dimensionless heat absorption coefficient.

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III. SOLUTION OF THE PROBLEM

Equations (9) & (10) represent a set of partial differential equations that cannot be solved in closed form. But, it can be reduced to a set of ordinary differential equations in dimensionless from that can be solved analytically. This can be done by taking in the following way.

$$u(\eta, t) = l_0(\eta) + \varepsilon e^{nt} l_1(\eta) + \dots \dots$$
(13)

$$\theta(\eta, t) = m_0(\eta) + \varepsilon e^{nt} m_1(\eta) + \dots \dots \tag{14}$$

Substituting equations (13) and (14) into equations (9) & (10), equating like terms and ignoring higher order terms, we can get the following pairs of equations for $(l_0, m_0) \& (l_1, m_1)$.

$$R_m l_0^{'''} + l_0^{'} + l_0^{'} - N_1 l_0 = -[N_1 + m_0 Gr]$$
⁽¹⁵⁾

Where prime denotes ordinary differentiation with respect to n. Without going into detail, the velocity and temperature

(20)

 $l_0 = 1, l_1 = 1, m_0 \rightarrow 0, m_1 \rightarrow 0 \quad as \quad \eta \rightarrow \infty$

distributions in the boundary layer are

$$R_{m}l_{1}^{'''} + [1 - nR_{m}]l_{1}^{''} + l_{1}^{'} - [N_{1} + n]l_{1} = -[n + N_{1}] - m_{1}Gr - AR_{m}l_{0}^{'''} - Al_{0}^{'}(16)$$

$$m_0 + \Pr m_0 - Q \Pr m_0 = 0 \tag{17}$$

$$m_1'' + \Pr m_1 - (n+Q)\Pr m_1 = -A\Pr m_0'$$
 (18)

The corresponding boundary conditions are

$$l_0 = U_p, l_1 = 0, m_0 = 1, m_1 = 1 \quad at \quad \eta = 0$$
⁽¹⁹⁾

$$u(\eta,t) = l_{0}(\eta) + \varepsilon e^{nt} l_{1}(\eta)$$

$$= [l_{01} + R_{m} l_{02}] + \varepsilon e^{nt} [l_{11} + R_{m} l_{12}]$$

$$= [(h_{4}e^{-z_{6}\eta} + h_{3}e^{-z_{1}\eta} + 1) + R_{m} (h_{7}e^{-z_{8}\eta} + h_{5}e^{-z_{6}\eta} + h_{6}e^{-z_{1}\eta})] +$$

$$\varepsilon e^{nt} \begin{bmatrix} (-h_{16}e^{-z_{10}\eta} + h_{14} + h_{9}e^{-z_{3}\eta} + h_{15}e^{-z_{1}\eta} + h_{12}e^{-z_{6}\eta}) + \\ R_{m} (-h_{27}e^{-z_{12}\eta} + h_{22}e^{-z_{6}\eta} + h_{23}e^{-z_{1}\eta} + h_{24}e^{-z_{8}\eta} + h_{25}e^{-z_{10}\eta} + h_{26}e^{-z_{3}\eta}) \end{bmatrix}$$
(21)

$$\theta(\eta, t) = m_0(\eta) + \varepsilon e^{\eta t} m_1(\eta)$$

= $\left(e^{-z_1 \eta}\right) + \varepsilon e^{\eta t} \left(h_2 e^{-z_3 \eta} + h_1 e^{-z_1 \eta}\right)$ (22)

The skin-friction coefficient and Nusselt number for this boundary layer flow can be defined as:

$$\tau = \left(\frac{\partial u}{\partial \eta}\right)_{\eta=0} \text{ and } Nu = -\left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=0}$$
(23)

IV. RESULTS AND DISCUSSION

The objective of the present paper is to investigate the effects visco-elastic Rivlin-Ericksen fluid flow past an semiinfinite vertical plate embedded in a porous medium with variable temperature and suction. In order to get a clear insight of the physical problem, the numerical computations have been carried out for various values of material parameters such as visco-elastic parameter Rm, heat absorption parameter Q, Grashof number Gr, Prandtl number Pr, time t, suction velocity parameter A, moving velocity parameter Up and an exponential parameter ε , which are of physical and engineering interest. The numerical results are displayed with the help of graphical illustrations in figures 1-13. The parameters Pr = 0.71, n = 1, Q = 1 are kept fixed throughout the discussion.

Fig.1 presents the variation of the velocity distribution across the boundary layer for different values of the plate velocity Up in the direction of the fluid flow. Although we have different initial plate velocities, the velocity increases to the constant value for given material parameters. Fig. 2 shows the effect of Prandtl number (Pr) on the velocity profiles. It is observe that the velocity decreases with increasing values of Prandtl number (Pr). It is of great interest to show how the time t effects on velocity and temperature. It is observed that velocity profile u increases strongly far field regime in Fig. 3, temperature profile θ increases strongly near and far field regime in Fig. 10. Fig.4 presents the variation of the velocity distribution across the boundary layer for different values of the plate velocity ε in the direction of the fluid flow. The velocity increases as ε is increasing.

Effect of heat absorption parameter on velocity is presented in fig.5. From this figure, it is noticed that velocity decreases with an increase in heat absorption parameter. Fig.6 exhibits the effects of visco-elastic parameter on fluid velocity for the flow past a cooled plate (Gr > 0). With the amplification of visco-elastic parameter, the velocity pattern for cooled plate gradually decreases on velocity components. It may be concluded that the energy due elastic property of the fluid increases the velocity and then gets dissipated. Grashof number studies the behavior of free convection and it is defined as the ratio of buoyancy force to viscous force. It plays an important role in heat and mass transfer technology. In this study, the results are discussed for the flow past an externally cooled plate (Gr > 0) and flow past an externally heated plate (Gr < 0). Fig.7 is depicted for positive values of the buoyancy parameter Gr which corresponds to the cooling problem. The cooling problem is often encountered in the cooling of nuclear reactors. It is experienced from Fig. 7 that the velocity rises as increasing values of Grashof number (Gr). Fig.8 presents the variation of the temperature distribution across the boundary layer for different values of the plate temperature ε in the direction of the fluid flow. The temperature increases as ε is increasing. Effect of heat absorption parameter on temperature is shown in fig. 9, from which it is concluded that the temperature decreases as heat absorption parameter increases. The effects of A on the temperature of the fluid are depicted through graph 11. The temperature decreases for increase in A. The effects of n on the temperature of the fluid are depicted through graph 12. The temperature increases for increase in n. Fig. 13 shows the effect of Prandtl number (Pr) on the temperature profiles. It is observe that the temperature decreases with increasing values of Prandtl number (Pr).

Effects of various parameters on skin friction and rate of heat transfer are presented in tables I-II. After knowing the velocity field, it is very important from a physical point of view to know the effect of viscoelastic parameter on resistive force or viscous drag. The resistive force or viscous drag on the surface of the body due to the motion of the fluid is known as the shearing stress or skin-friction coefficient. Table I depicts that on increasing thermal Grashof number Gr, visco-elasticity parameter Rm, Radiation absorption parameter Q, time t, Prandtl number Pr, suction velocity parameter A and Scalar constant ɛ, skin friction coefficient τ increases. From table II, it is observed that Nusselt number increases for increasing values of radiation absorption parameter Q, time t, Prandtl number Pr and suction velocity parameter A.























Fig. 7 Variation of u with η on for different values of Gr









Fig. 11 Variation of θ with η on for different values of A



Fig. 12 Variation of θ with η on for different values of n



Fig. 13 Variation of θ with η on for different values of Pr

Gr	Rm	Q	t	Pr	Α	3	Cf
1							1.1541
2							1.4386
3							1.7231
4							2.0076
	0.01						2.3082
	0.02						4.6163
	0.03						6.9245
	0.04						9.2326
		0.1					1.1184
		0.2					1.1216
		0.3					1.1249
		0.4					1.1285
			0.2				1.2755
			0.3				1.4096
			0.4				1.5578
			0.5				1.7217
				1			1.2603
				2			5.4156
				3			7.8720
				4			8.2912
					1		1.7435
					2		2.9224
					3		4.1012
					4		5.2801
						0.02	2.3082
						0.03	3.4622
						0.04	4.6163
						0.05	5.7704

TABLE I CF FOR DIFFERENT PARAMETERS

TABLE II NU FOR DIFFERENT PARAMETERS

Q	t	А	Pr	Nu
1				1.8174
2				2.2155
3				2.5328
4				2.8050
	0.1			1.4922
	0.2			1.5156
	0.3			1.5415
	0.4			1.5701
		0.1		1.7152
		0.2		1.7265
		0.3		1.7379
		0.4		1.7492
			0.71	1.8174
			1.00	2.3297
			2.00	3.9860
			3.00	5.5786

V. CONCLUSION

The governing equations for combined effects of heat absorption MHD on convective Rivlin–Ericksen flow past a semi-infinite vertical porous plate with variable temperature and suction was formulated. The plate velocity was maintained at a constant value and the flow was subjected to a transverse magnetic field. Numerical results are presented to illustrate the details of the flow and heat transfer characteristics and their dependence on the material parameters. It is seen that as Up increases, velocity increases. It is seen that the velocity decreases with increasing values of Pr. It is observed that velocity profile u increases strongly far field regime. It is observed the velocity increases as ε is increasing. It is noticed that velocity decreases with an increase in Q. It is observed the velocity decreases as Gr increases. It is noticed that the temperature increases as ε is increasing. It is concluded that the temperature decreases as Q. It is seen that the temperature decreases for increase in A. It is observed that the temperature decreases with increasing of Pr.

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