A Case Study on Non-Newtonian Viscosity of Blood through Atherosclerotic Artery

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Abstract - Present in the study of rheological character of blood flow and investigate the significance of non-Newtonian viscosity of blood through stenosed artery by assuming blood as Bingham Plastic and Casson’s fluid models. The results show that blood pressure increases very significantly in the upstream zone of the stenotic artery as the degree of the stenosis area severity increases. It is also shown that the non-Newtonian behaviour of blood has significant effects on the velocity profile of the blood flow and the magnitude of the wall shear stresses. It has been concluded in this paper that the Casson’s fluid model is more realistic in comparison to Bingham Plastic fluid model.

Key words: Non-Newtonian, Apparent Viscosity, Resistance to Flow, Bingham Plastic Fluid Model, Casson’s Fluid Model

I. INTRODUCTION

For many decades, cardiovascular disease has been one of the most severe diseases causing a large number of deaths worldwide each year, especially in developed countries. Most of the cases are associated with some form of abnormal flow of blood of stenotic arteries. In the presence of stenosis or “Atherosclerosis” (fig.a), the normal blood flow through the artery is disturbed resulting in blood recirculation and wall shear stress oscillation near the stenosis. The heart has to work harder and the blood cannot flow well, the narrowing region so as to enforce the blood circulation. If the heart works too hard and the blood cannot flow well, the narrowing region so as to enforce the blood circulation. If the heart works too hard and the blood cannot flow well, the narrowing region so as to enforce the blood circulation. If the heart works too hard and the blood cannot flow well, the narrowing region so as to enforce the blood circulation.

It is generally accepted that the blood, being a suspension of cells, behaves as a non-Newtonian fluid at low shear rate Charm and Kurland, (11); Hershey et al., (12) Whitmore, (13); Cokelet, (14); Lih, (15); Shakla et al. (16). It has been pointed out that the flow behaviour of blood in a tube of small diameter (less than 0.2 mm) and at less than 20sec−1 shear rate, can be represented by a power-law fluid Hershey et al., (12); Charm et al., (11). It has also been suggested that at low shear rate (0.1 sec−1) the blood exhibits yield stress and behaves like a Casson-model fluid Casson, (17); Reiner and Scott-Blair, (18). For blood flows in large arterial vessels (i.e., vessel diameter ≥1mm) Laburbara, (19), Jung (23), which can be considered as a large deformation flow, the predominant feature of the rheological behavior of blood is its shear rate dependent viscosity, and its effect on the hemodynamics of large arterial vessel flows has not been understood well. In this paper the effect of non-Newtonian viscosity of blood through stenosed artery has been investigated. The effect of stenosis on the resistance to flow, apparent viscosity and wall shear stress in an artery by considering the blood as a Bingham Plastic fluid and Casson’s-model fluids have also been determined. And to examine the effect of stenosis shape parameter, we considered blood flow through an axially non-symmetrical but radially symmetric stenosis such that the axial height of stenosis has much less than unity.
The resistance to flow from equation (11) using equations (10) is written as,
\[ \lambda = \frac{1}{R_0} \left( \frac{dP}{dz} \right) \frac{1}{R_0^2} \frac{d\theta}{dz} \frac{dQ}{dz} \]
(12)
where \( \theta \) is given by
\[ \theta = \frac{R_0^2}{R^2} + \frac{R_0^2}{R^2} \frac{dR}{dz} \left( \frac{1 - 6}{2} - \frac{1}{2} \right) \]
(13)
The shearing stress at the wall can be defined as;
\[ \tau_w = \tau_0 \left( \frac{dQ}{dz} \right) \frac{1}{R_0} \]
(14)

Casson’s Fluid Model: The Casson’s relation is commonly written as:
\[ \tau^2 = \tau_0^2 + (\mu_s)^2 \left( \frac{dQ}{dz} \right)^2 \]
if \( \tau \geq \tau_0 \)
(15)
\[ \frac{dQ}{dz} = 0 \]
if \( \tau < \tau_0 \)
(16)
where \( \tau_0 = \frac{R_0^2}{R^2} \frac{dR}{dz} \)
\( R \) is the radius of the plug-flow region, \( \tau_0 \) is yield stress, \( \tau \) is wall shear stress and \( \mu_s \) denotes Casson’s viscosity coefficient.
The Volume rate of flow using equation (16) is defined as,
\[ Q = \pi \rho \left( \frac{dQ}{dz} \right) \frac{1}{R_0^2} \]
(17)
By integrating equation (17), using equations (16) and (3) we have,
\[ Q = \pi \rho \left( \frac{dQ}{dz} \right) \frac{1}{R_0^2} \]
(18)
Equation (18) can be rewritten as;
\[ Q = \pi \rho \left( \frac{dQ}{dz} \right) \frac{1}{R_0^2} \]
where \( \theta = \frac{1}{R_0} \left( \frac{dQ}{dz} \right) \frac{1}{R_0^2} \),
with \( \gamma = \frac{R_0^2}{R} + \frac{1}{R} \)
From above equation pressure gradient is written as follows,
\[ \frac{dP}{dz} = \frac{dQ}{dz} \frac{1}{R_0^2} \frac{1}{R_0^2} \frac{1}{R_0^2} \]
(19)
Integrating equation (19) using the condition \( P = P_w \) at \( z = 0 \) and \( P = P_a \) at \( z = L \). We have,
\[ \Delta P = P_w - P_a = \frac{R_0^2}{R^2} \frac{dQ}{dz} \frac{1}{R_0^2} \frac{1}{R_0^2} \]
(20)
The resistance to flow (resistive impedance) is denoted by \( \lambda \) and defined as follows [Young, (4)],
\[ \lambda = \frac{R_0^2}{R^2} \frac{dQ}{dz} \]
(21)
The resistance to flow from equation (21) using equations (16) and (3) is written as,
\[ \lambda = \frac{1}{R_0^2} \frac{dQ}{dz} \frac{1}{R_0^2} \frac{1}{R_0^2} \]
(22)
where \( \theta \) is given by
\[ \theta = \frac{1}{R_0^2} \left( \frac{dQ}{dz} \right) \frac{1}{R_0^2} \frac{1}{R_0^2} \]
(23)
The shearing stress at the wall can be defined as;
\[ \tau_w = \frac{R_0^2}{R^2} \frac{dQ}{dz} \frac{1}{R_0^2} \]
(24)

V. RESULTS AND DISCUSSION

In order to have estimate of the quantitative effects of various parameters involved in the analysis computer codes were developed and to calculate the analytical results obtained for resistance to blood flow (\( \lambda \)) apparent viscosity and wall shear stress for normal and diseased system associated with stenosis due to the local deposition of lipids have been determine. Fig.2 reveals the variation of resistance to flow (\( \lambda \)) with stenosis size (\( \delta/R_0 \)) for different values of flow behavior index (n). It is observed that the resistance to flow (\( \lambda \)) increases as stenosis size (\( \delta/R_0 \)) increases. It is also noticed here that resistance to flow (\( \lambda \)) increases as flow behavior index (n) increases. It is seen from this graph that the wall shear stress (\( \tau \)) increases as value of flow behavior index (n) increases. As the stenosis grows, the wall shearing stress (\( \tau \)) increases in the stenotic region. It is also noted that the shear ratio given by (15) is greater than one and decreases as n decreases (n < 1). These results are similar with the results of [Shukla, et al., (16)]. It is also seen that the ratio is always greater than one and decreases as n decreases. For \( \delta/R_0 = 0.1 \) the increases in wall shear due to stenosis is about 37% when compared to the wall shear corresponding to the normal artery in the Newtonian case (n = 1), but for n = 2/3 this increase is only 23% approximately. However, for \( \delta/R_0 = 0.2 \), the corresponding increase in Newtonian (n = 1) and non-Newtonian (n = 2/3) cases are 95% and 56% respectively.

Fig.6 shows the distributions of resistance to flow (\( \lambda \)) with stenosis size (\( \delta/R_0 \)) for different values of stenosis shape parameter (m). It is seen from the figure that the resistance to flow (\( \lambda \)) is always greater than unity and increases as stenosis size (\( \delta/R_0 \)) increases and decreases as the stenosis shape parameter (m) increases. Maximum resistance to flow (\( \lambda \)) occurs at m = 2. i.e. in the case of symmetric stenosis. This result is therefore consistent with the observation of [Haldar, (2010)]. Fig.7 depicts the variation of resistance to flow (\( \lambda \)) with stenosis length (\( L/L_0 \)) for different values of stenosis shape parameter (m). As it is seen from the Fig.2, Fig.3 that the ratio is always > 1 and decreases as n decreases (n < 1). These results are similar with the results of [Shukla, et al., (16)]. It is also seen that the ratio is always greater than one and decreases as n decreases. For \( \delta/R_0 = 0.1 \) the increases in wall shear due to stenosis is about 37% when compared to the wall shear corresponding to the normal artery in the Newtonian case (n = 1), but for n = 2/3 this increase is only 23% approximately. However, for \( \delta/R_0 = 0.2 \), the corresponding increase in Newtonian (n = 1) and non-Newtonian (n = 2/3) cases are 95% and 56% respectively.

Fig.8 represents variation of apparent viscosity (\( \mu_s \)) for different values of yield stress

Sapna Ratan Shah
Fig. 9 shows the variation of wall shear stress with stenosis length (L/L0) for different values of stenosis shape parameter (m). We observe that the wall shear stress sharply increases as length of stenosis (L/L0) increases and decreases as stenosis shape parameter (m) increases. [Tandon, et al., (22)] have also noted the same results.

**VI. CONCLUSION**

In this paper, we have studied the effects of the stenosis in an artery by considering the blood as Bingham Plastic and Casson’s model fluids. It has been concluded that the resistance to flow and wall shear stress increases as the size of stenosis increases for a given non-Newtonian model of the blood. These increases are however, small due to non-Newtonian behaviour of the blood. It is noted that the non-Newtonian behaviour of the blood has very significant effect on the magnitude of wall shear stresses. The non-Newtonian behaviour has very significant effect on the magnitude of wall shear stresses. This results in a higher pressure to impel the blood passing through the narrowing channel.

**REFERENCES**


Sapna Ratan Shah
