

# Performance Investigation of Topologically Optimized Manipulator Link

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**Abstract -** Reducing weight of components in a manipulator is the need of robotic industry, not only to save cost by reducing the material but also to optimize the power consumption. Hence, topology optimization of manipulator links has wider scope to research and deliver the best possible solutions to design a system. The paper focus on investigating the performance of the link using topology optimization and dynamic modeling. In this work, topology optimization is utilized to generate optimum shape and size of link, finite element approach is adopted in finding the inertia of the given body, deflection with stress and followed by kinematic modeling. Further trajectory planning is done and Lagrange-Euler formulation is applied to find out required joint-torque values for each increment of motion.

**Keywords:** Robotic/Manipulator-link, Mass reduction, Joint-Torque reduction, Topology optimization, SIMP.

## I. INTRODUCTION

Research on joint torque minimization for manipulators has received considerable attention in recent years. For this aim, various methodologies are proposed using the concept of flexible manipulator or introducing the light mechanisms for links [1-5]. These methodologies involves complexities in the modeling as well as require the change in the control strategy of the manipulator. To overcome these issues, another method of reducing joint-torque is proposed in the presented paper. Here, the weight of the manipulator link is subject to mass reduction, which in turn reduces the inertial component, and hence the joint torque requirement. In order to reduce the weight of the manipulator-link, the help of topology optimization is taken. Topology optimization is a newly introduced method for material minimization in the design of structures and machine components. In this process, the designer specifies desired conditions such as amount of material to be removed, overall material domain, forces and other boundary conditions. The basic topology optimization method produces the optimal topology for these given conditions. It is a large-scale optimization of the material within the predefined material zone. Various topology methods are developed by the researchers, in which SIMP (Solid Isotropic Microstructure with Penalization), ESO (Evolutionary Structural Optimization) and Level Set methods are the largely applied methods [6]. These methodologies combine other small algorithms to capture the numerical instability in the solution [6, 7]. Finally, the basic topology is generated for other subsequent process towards manufacturing of optimized component.

In the presented paper an example of 1-DOF (degree of freedom) link is taken to show the reduction of joint torque using topology optimization. The manuscript is organized in following way. Section two describes the modeling of 1-DOF link. The topology optimization pprocedure of the link is elaborated in section three. The results and discussion for the reduction in joint torque is given in section four, and finally conclusion is drawn.

## II. MODELING OF 1-D ROBOTIC LINK

The application of topology optimization method on the manipulator link is illustrated on a 1-DOF link as shown in Fig.1. The kinematic, dynamic modeling, and trajectory planning is presented in this section.



Fig. 1 1-DOF manipulator link

### A. Kinematic and Dynamic Modeling

The frame assignment of the link is presented in Fig. 2. Here, Denavit-Hartenberg convention is used for the kinematic modeling [9]. Based on this frame assignment, the kinematic model of 1-DOF manipulator link is given in equation (1), in terms of the homogeneous transformation matrix  ${}^0T_1$ .

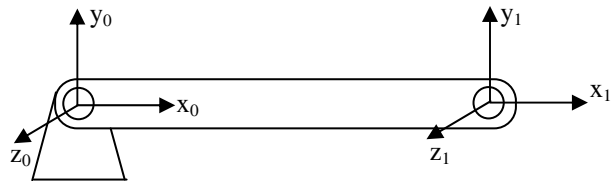


Fig. 2 Frame assignment of the link

$${}^0T_1 = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

The dynamic model of 1-DOF manipulator is prepared using Lagrange-Euler formulation [9]. The final dynamic model is represented in equation (2).

$$\tau = \frac{1}{3}mL^2\ddot{\theta} + \frac{1}{2}mgL\sin\theta \quad (2)$$

where,  $\tau$  is the joint torque,  $L$  is the link length, and  $g$  is the acceleration due gravity. The dynamic model is utilized to compute the joint torque for different angular motion of the link.

**B. Trajectory Planning**

In order to get smooth motion the trajectory planning is performed for the selected link. The constraint involved in the trajectory planning are, joint velocity and acceleration are assumed as zero at the initial and final point. The angular motion of the linked is assumed from zero to 180° in 5 seconds.

$$\theta = a_5t^5 + a_4t^4 + a_3t^3$$

$$\dot{\theta} = 5 \times a_5t^4 + 4 \times a_4t^3 + 3 \times a_3t^2$$

$$\ddot{\theta} = 20 \times a_5t^3 + 12 \times a_4t^2 + 6 \times a_3t$$

After employing the constraints, trajectory equations take following form [10],

$$\theta = 0.006 \times t^5 - 0.0754 \times t^4 + 0.2513 \times t^3 \quad (3)$$

$$\dot{\theta} = 5 \times 0.006 \times t^4 - 4 \times 0.0754 \times t^3 + 3 \times 0.2513 \times t^2 \quad (4)$$

$$\ddot{\theta} = 20 \times 0.006 \times t^3 - 12 \times 0.0754 \times t^2 + 6 \times 0.2513 \times t \quad (5)$$

where,  $t$  is time in seconds, and  $\theta$  is the joint motion angle. Based on the above trajectory, dynamic modeling is employed to observe the joint-torque requirement for each increment of the motion. In order to analyze the effect of mass reduction for the link, these calculations will be repeatedly performed and the significance of the mass reduction can be seen. As stated earlier, the mass reduction is performed using topology optimization method which is detailed in the next section.

**III. TOPOLOGY OPTIMIZATION**

Topology optimization is a powerful tool for the material, mass or volume minimization for structural or machine components [6, 7]. There are several methodologies available for topology optimization. In the present case the one of the methodology i.e. Structural Isotropic material with Penalization (SIMP) is utilized. According to SIMP, the topology optimization problem is defined in equation (6) [6, 7]

$$\left. \begin{aligned} \min_x : C(x) &= \sum_i^n x_i^p u_i^T K_0 u_i \\ \text{Subject to :} \\ V(x) &\leq V_f \\ 0 < x_{\min} &\leq x_i \leq 1, \quad i = 1, 2, 3, \dots, n \end{aligned} \right\} \quad (6)$$

where,  $C$  is the compliance value,  $K_0$  is the basic stiffness matrix,  $u_i$ 's are the element displacements vector,  $n$  is the total number of elements.  $V_f$  represents the required total

volume, and  $p$  is the penalization power, kept as three [6, 7].

For the present problem, the initial material domain is defined as a rectangular are with two holes to facilitate the joint assemble, as shown in Fig.3.

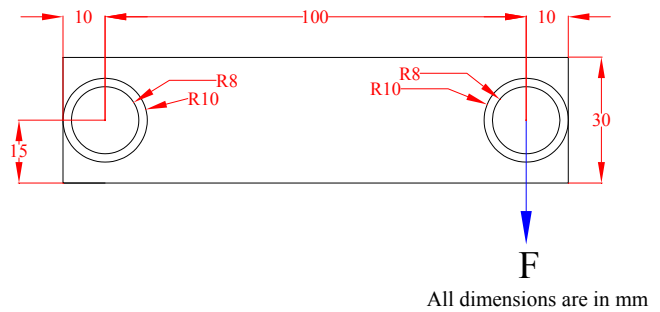


Fig. 3 Dimensional details of the manipulator link for topology optimization.

A MATLAB code it written for topology optimization process [8-10]. The results of topology optimization and joint-torque values are discussed in the next section.

**IV. RESULTS AND DISCUSSION**

Topology optimization process is applied for the compliance value as objective function and the volume fraction as the constraint value. Here the compliance values refers to the amount of strain energy stored in the component. Also, it is referred to the inverse of the rigidity of the structure [6, 7]. Hence, for the present case the compliance values should be as minimum as possible to provide the maximum flexibility to the link. However, due to high section modulus of the link, the maximum deflection of the mean remain in a negligible domain. Deflection values are also mention in the results. The volume fraction denotes the fraction of the material which has to be retained after the topology optimization comparing to the initial material domain. (Fig. 3). The values of compliance for different volume fraction is shown in Fig. 4 (a).

To analyze the effect of mass reduction of the flexibility of the link through the deflection, the maximum deflection of the link is also provided in Fig. 4(b). With the mass reduction, the strength also reduced hence it is also required to observe the maximum stress acting on the link. For this purpose, the maximum Von-Mises stress on the link is shown in Fig. 4(c). It can be seen, that for lower values of volume fraction, the compliance, and deflection decreases and Von-Mises stress increases.

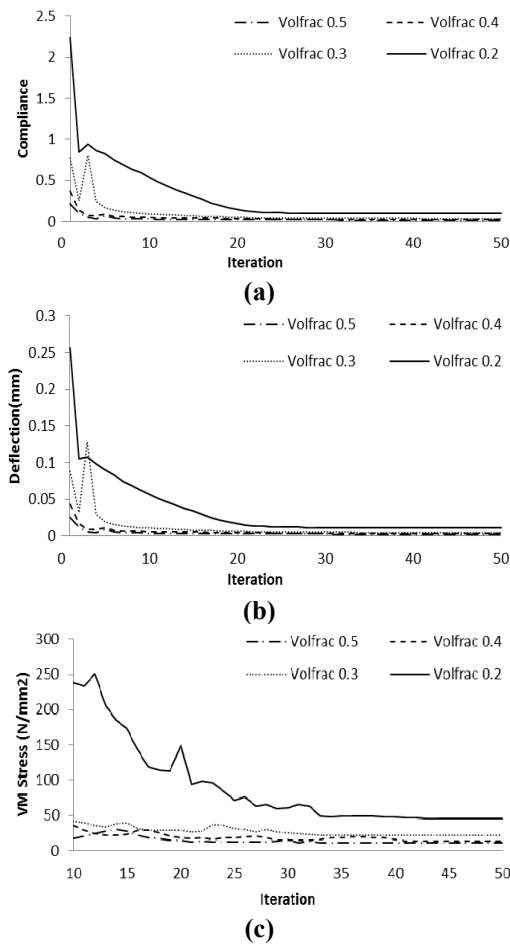


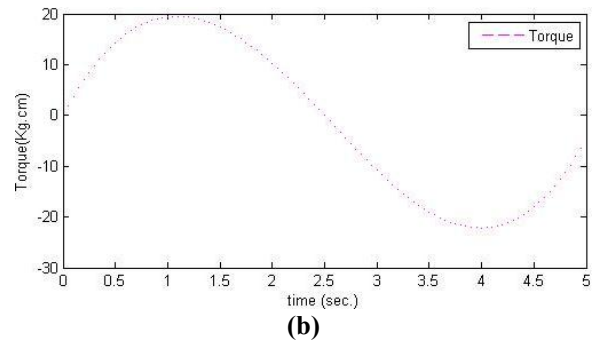
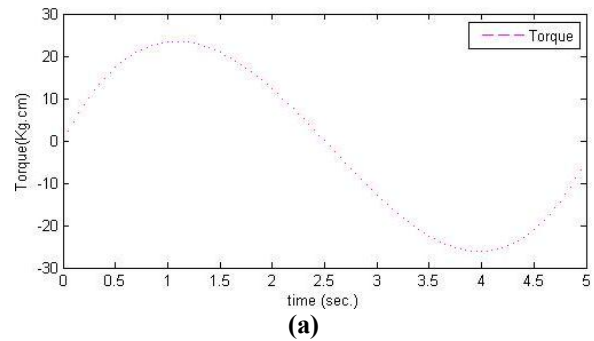
Fig.4 Different values of (a) compliance, (b) deflection, and (c) Von-Mises stress for the volume fraction values

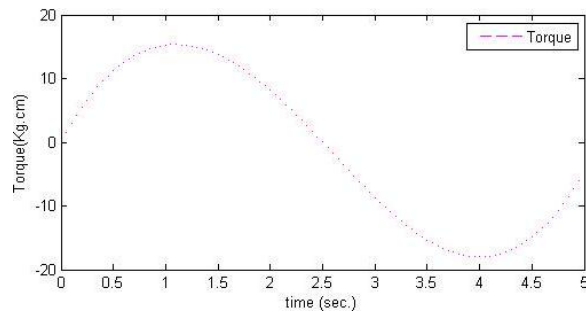
With the increasing number of optimization iteration, their values get stabilized. It can also be observed that when the volume fraction values is high, the stabilization occurs more quickly compare to lower values of volume fraction. It is because of the optimization characteristics.

To analyze and compare the different optimal topologies as each volume fraction, the optimal topologies with its corresponding compliance, deflection and stress values are given in Table 1. From this table, the optimal topologies can be referred with its properties. It can be observed that, with decreasing values of volume fraction, the compliance, deflection and Von-Mises stress increases. From the topology, it is seen that at lowest values of volume fraction, the topology tends to become a truss like structure. Using these results, the required joint-torque of the manipulator link can be computed. For this purpose, the dynamic model with trajectory is utilized and the joint-torque for the different topologies are found out for a payload of 10N. The obtained joint-torque variation curves are presented in Fig. 5.

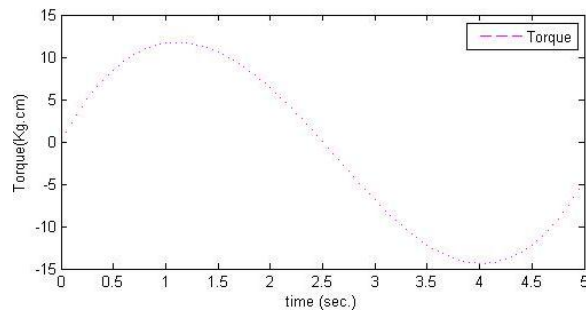
TABLE I OPTIMAL TOPOLOGIES FOR VARIOUS VOLUME FRACTION VALUES

Volume fraction & weight (gm)	Optimal topology
0.5 82.67	
	C*:0.0196, D:0.00231, S: 10.62
0.4 66.136	
	C: 0.0259, D:0.00303, S: 13.79
0.3 49.60	
	C: 0.0392, D:0.00463, S: 21.88
0.2 33.07	
	C: 0.0971, D:0.01109, S: 45.28
*C: Compliance, D: Max. Deflection (mm), S: Max. Von Mises Stress (N/mm <sup>2</sup> )	





(c)



(d)

Fig.5 Joint-Torque curves for volume fraction (a) 0.5, (b) 0.4, (c) 0.3, (d) 0.2

From Fig. 5, the different joint-torque requirement for the different volume fraction value can be observed. With decreasing volume fraction (or mass) value, the joint torque values also decreases. The trend or patterns of the joint torque value remains same in all cases. The presented analysis is helpful to select the volume fraction values based on the offered values of compliance, deflection and stress.

## V. CONCLUSION

Present work focuses upon the effect of mass reduction of manipulator link on the joint torque values. For this purpose a combined methodology of manipulator modeling with topology optimization scheme was proposed. The methodology was illustrated using a 1-DOF manipulator. It is observed that by reducing the mass of link by 80% the maximum deflection reaches up to  $11.09\mu m$ , and by reducing the mass by 50% the maximum deflection reaches up to  $2.31\mu m$ . Correspondingly the required joint-torque value also reduces by a large extent without having any effect on its trend. Hence, the methodology is helpful to reduce the inertial load or dynamic loading on the manipulator, and hence the energy requirement. In addition, by reducing the inertial load the accuracy of the manipulator can be further increased.

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