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A Simple Tool for Solving Queuing Model with Erlang Distribution

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Abstract **- In this paper we use the online tool to find the performance measures of the steady-state behavior of the queuing model $E_k/E_m/1$ where inter arrival and service times are Erlang distributions with parameters k and m respectively. The suggested tool has a simple graphical interface, based on a recently published simple recurrence method[1], it provides the performance measures of the queuing system (i.e., average number in the system, average waiting time, the system utilization, probability of loss, probability of waiting time), also the steady-state distribution of the system.**

I. INRODUCTION

The pioneer investigator of queuing theory was the Danish mathematician A.K. Erlang. Since that time, the well-known Erlang's *B*-formula for the probability loss in the queuing model $M/M/N/0$, i.e., *N* server queuing system with no waiting space, where the arrival process is Poisson and the service time distribution is exponential, provided a good mathematical tool for planning capacity and evaluation of performance measures in the classic telephone networks. This is explained by the facts that (i) the flows of information in such networks were well described by the stationary Poisson arrival process and (ii) distribution of the number of busy servers in the $M/G/N/0$ system is insensitive with respect to the service time distribution under the fixed mean service time. The assumption made by A.K. Erlang that the service time distribution is exponential (which is not true in real-life systems) was not rough. However, the flows in the modern telecommunication networks have lost the nice properties of their predecessors as like in the old classic networks. In opposite to the stationary Poisson flow, the modern real-life flows are non-stationary, group and correlated.

Consider a queuing model $E_k/E_m/1$ where inter arrival and service times follow Erlang distributions with parameters k and m respectively. Arriving customers are served under the discipline First-come First-served (FCFS). A state of system is denoted by (*n*, *i*, *j*) where *n* is the number of customers in the service or in the waiting room, $n \geq 0$, i represents the phase of the customer in the inter arrival time distribution and j the service time distribution where $1 \leq i \leq k,\ 1 \leq j \leq m$.

The inter arrival time distribution and its representation of dimension *k* are denote by $F(\cdot)$ and (β_1, S_1) respectively, where the $\beta_1 = (1, 0, \ldots, 0)$ is a 1×*k* row vector and S_1 is a square matrix of dimension *k*, i.e., The mean arrival rate is given by λ/k . The distribution function is given by

$$F(t) = 1 - \beta_1 exp(S_1 t)\mathbf{1}$$

$$S_1 = \begin{bmatrix} -\lambda & \lambda & & & \\ & -\lambda & \lambda & & \\ & & \ddots & \ddots & \\ & & & -\lambda & \lambda \\ & & & -\lambda & \lambda \end{bmatrix}$$

Where 1 is a column vector of all entries equal to 1.

In spite of several literatures available to find the solution of multi-server queues, queuing theorist, researcher and practitioners still find it challenging task to obtain an accurate solution in short for the ordinary queue $G/M/c$. We use the web based designed tool with a graphic interface for solving the steady state behavior of $E_k/E_m/1$. The tool is simple and easy to use, also any performance analyst can run this through internet access.

This paper work is organized as follows: Section II describes the capabilities of the suggested tool, and Section III provides the computed results.

II. TOOL DESCRIPTION

The suggested tool provides the solution in exact form to the model $E_k/E_m/1$. The tool is portable as possible and can implement as web application. The tool includes (i) some PHP code for retrieving the submitted parameter values and for presenting the results, (ii) JavaScript function to enhance the user interface and (iii) the use of the graphical library Protovis [2] to build a dynamic chart.

Steady State Solution
1) Type in the parameters of the queue

C [8] Number of Server

N [16] Maximum number in system (buffer + server)

2) Select the inter-arrival time

by: (•) its mean and coefficient of variation ( ) or a specific Coxian distribution

t_a [10] Mean time between arrivals

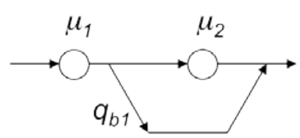
cv_a [8] Coefficient of variation

3) Select the service time

by: (•) its mean and coefficient of variation ( ) or a specific Coxian distribution

t_s [15] Mean service time

cv_s [8] Coefficient of variation



Services follow a Coxian distribution with 2 stages with

μ_1 = [0.133], μ_2 = [0.001] *and* q_{b1} = [0.992].

III. COMPUTED RESULTS

Parameters of the queue

C = 8,

N = 16

Inter-arrivals time: t_a = 10 and cv_a=8

description by a Coxian distribution:

τ_1 = 1.000
λ_1 = 0.200
r_{b1} = 0.992
τ_2 = 0.000
λ_2 = 0.002
r_{b2} = 1.000

Service time: t_s = 15 and cv_s = 8

description by a Coxian distribution:

σ_1 = 1.000
μ_1 = 0.133
q_{b1} = 0.992

σ_2 = 0.000
μ_2 = 0.001
q_{b2} = 1.000

Approximate results:	
Mean number in system	$\bar{Q} = 1.5196$
Mean waiting time	$\bar{W} = 0.2520$
Mean throughput	$\bar{X} = 0.09963$
System utilization (in %)	$U = 18.68$
Probability of loss	$P_{loss} = 0.00368$
Probability of waiting	$P_{wait} = 0.01700$

Steady-state distribution for the number of customers in the system from n= 0 to n= 16

n	p(n)
0	0.50871
1	0.09173
2	0.11923
3	0.11221
4	0.08068
5	0.0467
6	0.02259
7	0.00936
8	0.00339
9	0.0014
10	0.00067
11	0.00038
12	0.00026
13	0.00022
14	0.00022
15	0.00032
16	0.00194

IV. CONCLUSION

The solution obtained by the tool includes the performance measures such as average number in the system, average waiting time, system utilization, probability of loss, probability of i. Also it provides the steady state distribution of number of customer in the system. The tool suggested can solve the queue with unrestricted or restricted buffer. In case of a finite buffer customers arriving when the buffer is full are assumed to lose.

REFERENCES

[1] A. Brandwajn and T. Begin, “A recurrent solution of $Ph/M/c/N$ -like and $Ph/M/c$ -like queues,” Journal of Applied Probability, Vol. 49, No. 1, March 2012.

[2] M.Bostock and J.Heer, “Protovis: A graphical toolkit for visualization”, IEEE Transactions on Visualization and Computer Graphics, Vol. 15, Nov 2009.