

Minimizing the Delay Time of Data Transfer in Access Networks Using Queue Models

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Abstract - Modeling and performance evaluation are becoming critical issues in access networks. The upstream channel of cable access network regulated by request grant mechanism can be viewed as a two stage tandem queue. The intention of this paper is to divide the capacity of the upstream data among the two stations to ensure minimum delay time by considering the first node as M/M/1 queue and second node as M/M/1 retrial queue.

Keywords: Two stage tandem queue, M/M/1 queue, M/M/1 retrial queue, Access networks

I. INTRODUCTION

Emerging access networks like CATV, RAN and IP – CAN has witnessed rapid growth and technological innovations in the field of computer communications. Cable television (CATV) is a unidirectional medium carrying broadcast analog video channels to the most customers possible at the lowest possible cost to the CATV service provider. Since the introduction of CATV more than 50 years ago, little has changed beyond increasing the number of channels supported. To deliver data services over a cable network, one television channel (50 to 750 MHz range) is allocated for downstream traffic to homes and another channel (5 to 47 MHz band) is used to carry upstream signals. When upstream channels are used on the cable network, they typically occupy lower frequencies that are subject to noise. In addition, the typical system uses TDMA, so users must contend for access to time slots. As more people access the network, performance drops. Some systems are so noisy that providers require users to use dial-up connections for upstream data.

In the context of the cable access network, the upstream channel is the service system and the amount of data that users want to transmit is the service demand. When the capacity of the upstream channel is limited queue is formed. Queue causes delay and affects the quality of service provided to the user. The upstream channel of cable access network regulated by request grant mechanism can be viewed as a two stage tandem queue. When a user wants to transmit data, it first joins the request queue where it waits until its request gets granted. Once granted, the user moves to the data queue and waits until its data gets transmitted. The data queue is invisible, in the sense that packets are not actually lined up in the queue. Instead, the users hold their packets until they are allowed to actually transmit these. The total

service capacity for both queues is equal to the capacity of the upstream channel. The division of upstream capacity among the two queues will determine the delay experienced by the users at each of the two stages.

Queue models characterize these kinds of networks considered and the behavior can be studied in detail. The existing literature includes the investigation of retrial phenomenon in communication systems [1, 4, 7, 8, 9 and 12]. Modeling of a computer network, tandem queue has been pursued by many people, for example Resing et al [10], C.-K Toh [3] and in number of references cited there in. The rest of the paper is organized as follows. Section II explains the queuing model of the upstream data transfer. Numerical example to divide the frequency between the stations is explained in section II followed by conclusion in section III. Finally the references are listed in section IV.

II. MATHEMATICAL MODEL

Consider a two station tandem queue as shown below. Let arrival to S_1 be poisson with rate λ and the service times at S_1 is exponentially distributed with parameter $\mu_1 > \lambda$. As there is single server at S_1 we get M/M/1 queueing system at S_1 . Since the departures of S_1 is also Poisson with rate λ by equivalence property, the input to S_2 is poisson with rate λ . For access networks the second station with single server is considered as a retrial M/M/1 queue that is if a customer coming from S_1 finds the server in S_2 free he immediately occupied it and leaves the system after service. On the other hand if he finds the server busy upon arrival is obliged to leave the service area, but he repeats his demand after an exponential time with retrial rate ν . Let primary customer arrival rate be λ' such that $\nu + \lambda' = \lambda$. Let the service time at S_2 be exponential with parameter $\mu_2 > \lambda$. We also assume that inter arrival periods, service times and retrial times are mutually independent.

Let N_1 be the number of customers at S_1 and N_2 be the number of customers at S_2 at any time then the joint probability

$P[N_1 = n_1, N_2 = n_2] = P[N_1 = n_1] P[N_2 = n_2]$ by Burke's theorem. Also the delay experienced by the user is the sum of delay in S_1 and delay in S_2 . Also $\mu_1 + \mu_2$ equals the upstream capacity.

At S_1 , let P_n denote the probability of n customers in the system in steady state. The QBD matrix is

$$Q = \begin{pmatrix} -\lambda & \lambda & & & & \\ \mu & -(\mu + \lambda) & \lambda & & & \\ & \mu & -(\mu + \lambda) & \lambda & & \\ & & \mu & -(\mu + \lambda) & \lambda & \\ & & & \mu & -(\mu + \lambda) & \lambda \\ & & & & & \ddots \end{pmatrix}$$

given by

The limiting distributions are $P_n = \left(\frac{\lambda}{\mu_1}\right)^n \left(1 - \frac{\lambda}{\mu_1}\right)$ $n \geq 0$ and the delay time is $\frac{1}{(\mu_1 - \lambda)}$.

At S_2 , let $C(t)$ denote the number of busy servers and $N(t)$ denote the number of customers in the orbit and X be the process $\{C(t), N(t); t \geq 0\}$. Under above assumptions X becomes a regular continuous time Markov chain with state space $S = \{0, 1\} \times \{0, 1, 2 \dots \infty\}$.

By ordering the states as (0, 0), (1, 0), (0, 1), (1, 1)... we can express its QBD matrix as,

$$Q = \begin{bmatrix} A_0^{(0)} & A_0^{(1)} & 0 & 0 & \dots \\ A_1^{(-1)} & A_1^{(0)} & A_1^{(1)} & 0 & \dots \\ 0 & A_2^{(-1)} & A_2^{(0)} & A_2^{(1)} & \dots \\ 0 & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Where $A_0^{(0)}, A_0^{(1)}, A_j^{(-1)}, A_j^{(0)}, A_j^{(1)}$ are all 2×2 matrices given as below.

$$\begin{aligned} A_0^{(0)} &= \begin{pmatrix} -\lambda_1 & \lambda_1 \\ \mu_2 & -\lambda_1 - \mu_2 \end{pmatrix} \\ A_0^{(1)} &= A_j^{(1)} = \begin{pmatrix} 0 & 0 \\ 0 & \lambda_1 \end{pmatrix} \\ A_j^{(-1)} &= \begin{pmatrix} 0 & v \\ 0 & 0 \end{pmatrix} \\ A_j^{(0)} &= \begin{pmatrix} -\lambda_1 - v & \lambda_1 \\ \mu_2 & -\lambda_1 - \mu_2 \end{pmatrix} \quad \text{for } j = 1, 2, 3, \dots \end{aligned}$$

The limiting distributions are given by $P_{0j} = \frac{\rho^j}{j! \mu^j} (1 - \rho)^{1 + \frac{\lambda}{\mu}} \prod_{k=0}^{j-1} (\lambda + k\mu)$ $j \geq 0$ and

$$P_{1j} = \frac{\rho^{j+1}}{j! \mu^j} (1 - \rho)^{1 + \frac{\lambda}{\mu}} \prod_{k=1}^j (\lambda + k\mu) \quad j \geq 0.$$

The k^{th} moment is given by

$$M_k^i = \begin{cases} (1 - \rho) \left[\frac{\rho}{\mu(1 - \rho)} \right]^k \prod_{j=0}^{k-1} (\lambda + j\mu) & i = 0 \\ \rho \left[\frac{\rho}{\mu(1 - \rho)} \right]^k \prod_{j=1}^k (\lambda + j\mu) & i = 1 \end{cases} \quad \text{where } \rho = \frac{\lambda}{\mu_2}$$

Also, the delay time is $\frac{M_1^0 + M_1^1 + M_0^1}{\lambda_1}$.

Thus the total delay time in the system is

$$W_s = \frac{1}{\mu_1 - \lambda} + \frac{\lambda_1}{v\mu_2} + \frac{\lambda_1(\lambda_1 + v)}{\mu_2 v(\mu_2 - \lambda_1)} + \frac{1}{\mu_2}$$

A.Numerical Example

Let upstream signal strength be 45 MHz's Find the allocation of this strength between two servers such that the waiting time of a customer is minimum. Assume that

Case 1: Out of 22 customers, who get their grant in an hour, 7 customers directly transmit their data and 15 can transmit only after repeated retrials.

Case 2: Out of 15 customers, who get their grant in an hour, 5 customers directly transmit their data and 10 can transmit only after repeated retrials.

TABLE I SOLUTION: CASE 1 & CASE 2

Frequency in station 1	Frequency in station 2	Waiting Time	Frequency in station 1	Frequency in station 2	Waiting Time
23	22	1.10	16	29	1.06
24	21	0.60	17	28	0.57
25	20	0.45	18	27	0.40
26	19	0.37	19	26	0.32
27	18	0.33	20	25	0.28
28	17	0.31	21	24	0.25
29	16	0.31	22	23	0.23
30	15	0.31	23	22	0.21
31	14	0.32	24	21	0.20
32	13	0.34	25	20	0.20
33	12	0.38	26	19	0.20
34	11	0.45	27	18	0.20
35	10	0.57	28	17	0.20
36	9	0.80	29	16	0.21
37	8	1.53	30	15	0.22
			31	14	0.23
			32	13	0.25
			33	12	0.27
			34	11	0.30
			35	10	0.35
			36	9	0.42
			37	8	0.55
			38	7	0.79
			39	6	1.54

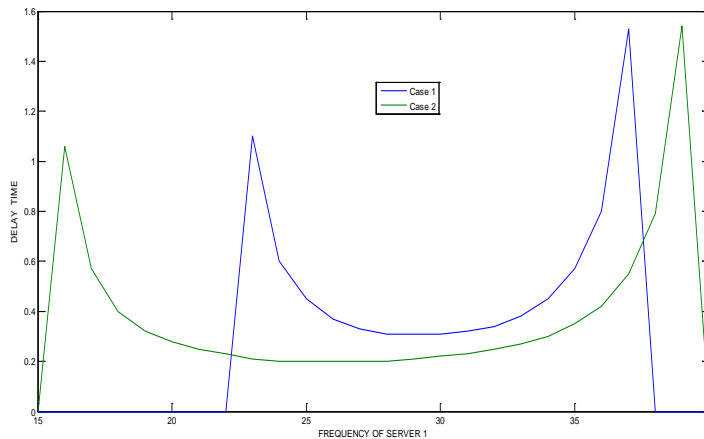


Fig.1 Frequency of server

It is clear from the graph that for case 1 the minimum waiting time is 0.31 and the frequency division range from 28 to 30 for server 1 and 17 to 15 for server 2. Similarly to achieve the minimum waiting time of 0.20 for case 2, the frequency division range from 24 to 28 for server 1 and 21 to 17 for server 2

III. CONCLUSION

Based on the incoming customers the capacity of the upstream data is divided among the two stations to ensure minimum delay of data transfer using queue models is studied. This will prohibit the loss of data and give quality of service to the customers.

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