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Analysis of Functionally Graded Cylinders for Different Gradation Law under Coupled Thermo-Mechanical Loading

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Abstract - **Functionally graded materials are materials with tailored properties in one or more directions. This paper analyzes functionally graded cylinders with variation of properties according to various gradation laws available in literature of functionally graded materials. These gradation laws determine response of material under different loading and boundary conditions. The whole analysis is carried out in ABAQUS. USDFLD subroutine has been used for varying the material properties at elemental level. FGM cylinder under analysis has been loaded with coupled thermo-mechanical load where one surface of the plate is loaded with mechanical force and a temperature gradient is provided over the thickness of the cylinder. Another surface is free from any kind of mechanical forces. On application of force and thermal gradient maximum stresses generated and the maximum nodal temperature observed in the cylinder is compared with the failure limits considering factor of safety for the working conditions.On the basis of maximum stress observed and the corresponding nodal temperature using different gradation laws the application of these laws are justified.**
Keywords: **USDFLD, Functionally Graded Materials, ABAQUS, Finite Element Analysis**

I. INTRODUCTION

Functionally graded materials (FGM) are heterogeneous materials which have gradation of various material properties in certaindirections. FGM were developed by Japanese scientists[1], [2] are designed for some specific kind of applications such as in aerospace engineering, other engineering fields like vibration control and high temperature applications, photonic devices like photodetectors and solar cells[3]. FGM are different from traditional composite materials which fail by delamination and stress concentration under the action of thermal load and mechanical load[4]. FGM overcomes this problem of delamination and stress concentrations due to smooth gradation of material properties in different directions instead of giving sudden variation in material property. Different materials in composites have sudden variation in properties so two or more materials cannot cope up with the properties of each other thus they shows sign of delamination and stress concentrations. Some of the commonly used FGM are ceramic metal composites which are capable of operating in high temperature range because ceramics provide better thermal resistance and metal phase provide the guarantee of structural integrity as well as provide toughness to the structure[5], [6].Cost of fabrication of FGM materials are quite high[7] as compared to other conventionally used materials but their advantages of various special properties which other materials cannot provide, diminishes the cost factor of FGM. Analysis of FGM is a typical process because of variation of material properties at every increment of distance with respect to the element chosen in any direction. As material properties changes so we cannot assume this whole material as a homogeneous material and thus analysis becomes difficult. For such analysis various techniques have been used earlier. One of the most used techniques is the variation of material properties using step gradation[8]–[10].

In this step gradation technique homogeneous layers of very small thickness are assumed and these layers are having uniform material properties over the entire thickness with properties matching with the mid line properties of the layer, but this technique is not sufficient to provide accurate solution. The accuracy of this technique depends upon the thickness of the homogeneous layer assumed. More fine the homogeneous layer more will be the accuracy of the solution. Another technique used for the analysis is variation of material properties at gradation points [11]. This is done with the help of subroutines which is a user defined material model called UMAT.

In [12] the author has used ABAQUS tool with the incorporation of UMAT for variation of properties for the calculation of crack tip field in a cracked FGM plate. This variation of material properties using ‘user defined material field’for graded elements provides better accuracy as compared to analysis using homogeneous layer method[13], hence we proceed with the technique of variation of material properties for graded elements, having better accuracy in comparison to homogeneous layer method. In the present problem the analysis of FGM cylinder is carried out with coupled thermo-mechanical load acting over it. For analysis the ABAQUS tool with the USDFLD code programmed in Fortran is used.

II. GRADATION OF MATERIAL PROPERTY

Literature available discusses various types of gradation laws using which properties of FGM are varied. Different gradation laws have different advantage over the application of FGM. Various gradation laws are as follows:

A. Exponential Law[4]

This law is particularly used in analyzing problems related to fracture mechanics [11], [13]. The material properties in thickness direction are specified according to formula:

$P(z)=P_t*e^f$

Where,

f	=	$(1/h)*[ln(Pb/Pt)]*(z+h/2)$
$P(z)$	=	Young's modulus variation with the thickness
P_t	=	Young's modulus of the top Layer
P_b	=	Young's modulus of bottom Layer
h	=	Maximum thickness in z-direction
$V(z)$	=	Volume fraction
n	=	Volume fraction exponent
z	=	Coordinate axis 3
h	=	Distance in direction of coordinate axis 3
x	=	Coordinate axis 1
y	=	Coordinate axis 2
r_o	=	Outer Radius
r_i	=	Inner Radius
r	=	Radius at any point

B. Power Law [4]

Power law is used for analysis of FGM shells/plates with uniform thickness. The variation of material properties using power law can be specified[14] as:

$P(z)=P_b*V(z)+P_t*[1-V(z)]$

$V(z)=[1-(r-r_i)/((r_o-r_i)]^n \quad ,(n\geq0)$

For the value of $n=\infty$, $P(z)=P_t$and at $n=0$ $P(z)=P_b$, meaning that once material property becomes equal to material property of top layer which can be either ceramic dominated or metal dominated and for the second case material property becomes equal to bottom layer material property. The graph showing the variation of dominating character of various materials on changing value of 'n' is shown in Fig.1 as:

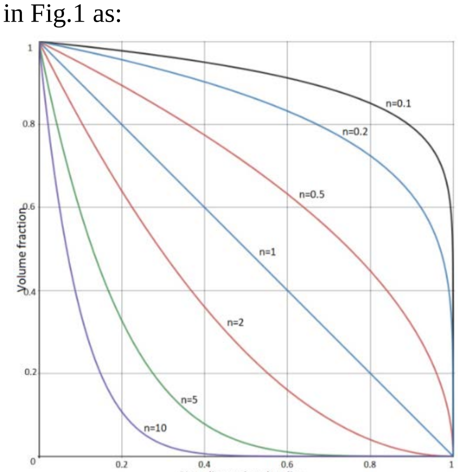

Fig. 1Variation of volume fraction versus non dimensional radius

C. Sigmoid Law [13]

Power law and exponential law both gives us the gradation of material properties of FGM, but both have the disadvantage of appearance of stress concentration at the interface. It appears like material is continuous but is rapidly changing. Another Law which overcomes the problem of both Power Law and Exponential law is used called Sigmoid Law.

III. FORMULATION

The problem being analyzed has variation of material properties as per power law defined in the gradation types. This variation is achieved by defining material properties at nodal points which is made possible with the help of user defined program coded in FORTRAN which is input as an another tool of ABAQUS. The gradation of the cylinder under analysis here is in through the thickness direction only. Dimensions of the cylinder arer$_i$=0.02m, r_o=0.04m. and height of the cylinder is 0.1m. Thickness of the cylinder varies from 0.02m to 0.01mBy varying the thickness of the cylinder efforts have been made to reduce the mass of the cylinder.FGM used here is made up of two different materials namely ZrO_2 and Ti-6Al-4V with the material properties as mentioned in Appendix 1. All the properties of the FGM such that specific heat, coefficient of thermal expansion, young's modulus, density, and conductivityis varied according to the power law as defined earlier. This variation of various material properties is made possible with the help of USDFLD code written in FORTRAN and which is input with ABAQUS tool. This code provides variation of properties at elemental level. The USDFLD code for the analysis is shown in Appendix 2. The mechanical load that acts as pressure is $2.094x10^6$ N/m^2 and the temperature that acts is of 1682.73 K and another surface is over temperature of 230 K thus providing a thermal gradient. All these values are from the design parameters of a low bypass turbofan engine in a jet trainer aircraft [15]. The geometric construction of the cylinder is shown in Fig. 2 as:

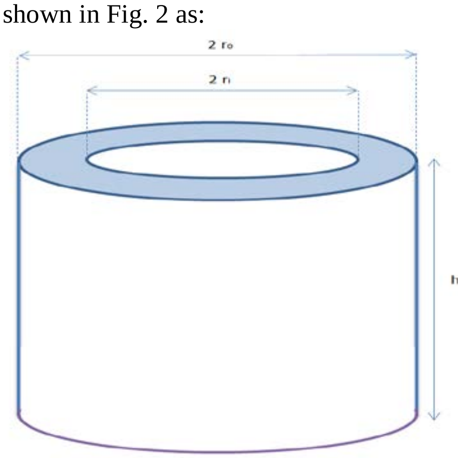

Fig. 2 Geometric construction of the structure

The mechanical load in form of pressure and thermal load acts on the internal walls of the cylinder. For the boundary conditions of the problem the structure is restricted to move in the z direction as specified in Fig. 3. The figure showing action of mechanical and thermal forces along with the boundary conditions is represented as Fig. 3 :

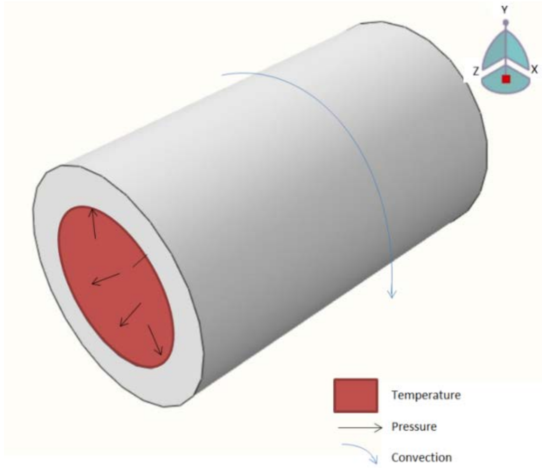

Fig. 3 Loading and boundary conditions of the structure

The analysis of the problem has been done using C3D8RT coupled temperature displacement element type.

IV. RESULTS AND DISCUSSION

The cylinder is analyzed for the varying value of the volume fraction index ‘n’ of the cylinder. Initial value of thickness is kept at 0.02m and then is further decreased to optimize the cylinder from mass point of view. Specified loading and boundary conditions are applied over the boundary of the structure. The 3 dimensional plot containing information about the maximum stress, temperature that is obtained in the structure and the mass of the structure for a definite thickness and with varying value of the volume fraction exponent ‘n’ is plotted in the Fig.4:

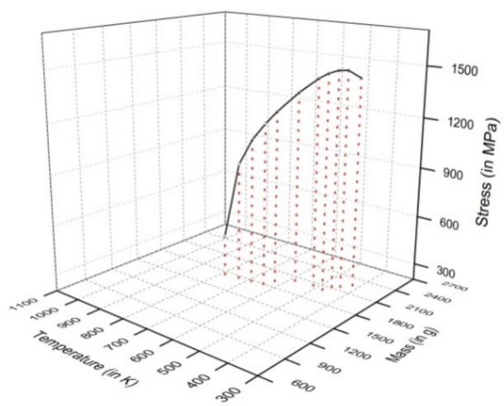

Fig. 43 Dimensional Plot for Plate Thickness of 0.02m

As is clear from the Fig. 4 that maximum value of stress and temperature for this plot are within the safe limit for the FGM. On varying the value of ‘n’ we have obtained this graph and corresponding change in mass is observed. We can proceed towards optimizing the results by decreasing the thickness of the cylinder So 3 dimensional plot for the thickness of the cylinder equal to 0.0175m is shown in Fig. 5 as:

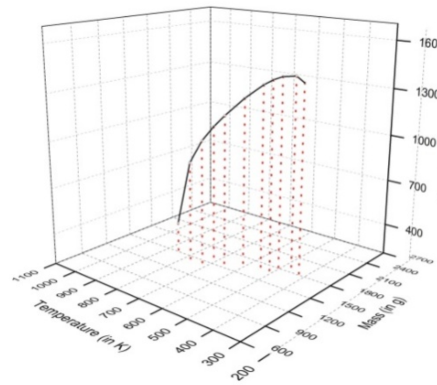

Fig. 5 3 Dimensional Plot for Plate Thickness of 0.0175m

These values are also lying between the safe limit for the volume fraction index of 3. So we proceed for the more less thickness cylinder which will further optimize the structure. So for the thickness of the cylinder equal to 0.015 m the 3 dimensional plot is drawn in Fig. 6 as:

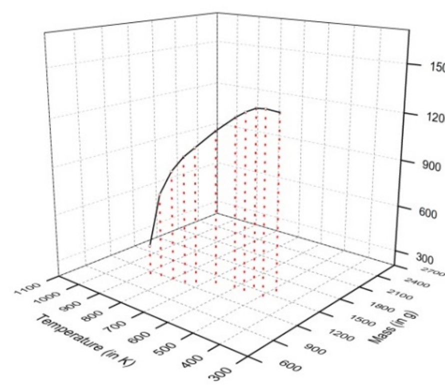

Fig. 6 3 Dimensional Plot for Plate Thickness of 0.015m

As is clear from the figure mass of the structure is decreasing with the decreasing thickness so this is optimizing the structure from mass point of view keeping the maximum value of stress and temperature within the safe limit for the FGM. Further optimization is done for the plate thickness of the cylinder equal to 0.0125m. The 3 dimensional plots for the same is shown in the Fig. 7 as:

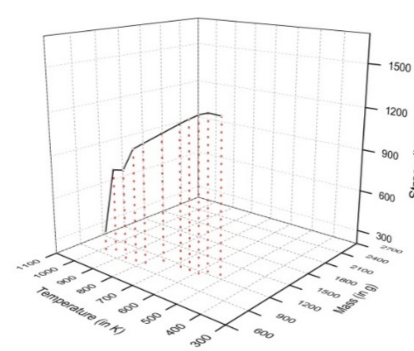

Fig. 7 3 Dimensional Plot for the Plate thickness of 0.0125 m

The graph drawn above for the FGM again shows that the values for the maximum stress and temperature for the volume fraction index ‘n=3’ are lying within the safe limit. The corresponding values are mass=966g, stress= 990 MPa, temperature=825K. We further decrease the thickness of the plate of cylinder from 0.0125 m to 0.01m. The corresponding 3 dimensional plot is shown in Fig. 8 as:

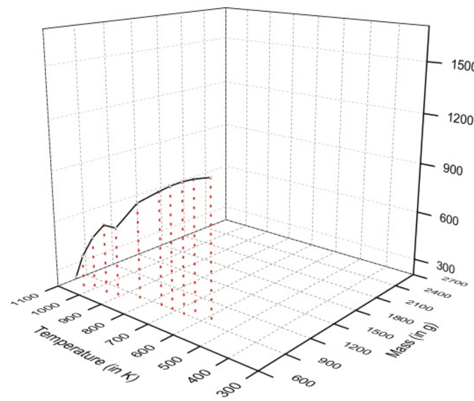

Fig. 8 3 Dimensional Plot for Plate Thickness of 0.01m

Now this graph gives us the final value of the mass for the plate thickness of 0.01m. For the volume fraction index ‘n=3’ the mass of the plate here is 737 g and the stress obtained is 627 MPa for the corresponding temperature value of 932 K. So ultimately this has again safe working limits for stress and temperature and this can be compared with the data available in Appendix 1. This optimization technique has led to a saving of 1023 g for the entire structure. When the structure for the thickness of 0.01m is entirely composed of Ti-6Al-4V the maximum allowable stress value is 1019 MPa and the corresponding temperature value is 558 K so this leads to failure ultimately. And on the other hand when volume fraction index ‘n’ becomes infinity the structure is totally made up of ZrO_2 and here the temperature value obtained is 1053 K and the stress is 263 MPa so this also fails. Ultimately FGM is used which don’t fail under these type of loading conditions and provide balance for the thermal and mechanical loading conditions. This is obtained for the volume fraction index of 3 here.

V. CONCLUSION

Present work has successfully concluded to have established the numerical solution and optimization technique of the coupled thermo-mechanical load acting over the cylinder. 3 dimensional plots have been used to obtain information about the optimization strategy. The above study and the optimization strategy has led to conclusion that:

1. The cylinder with single material cannot be used as it won’t be able to withstand the thermal or mechanical load and thus we would need an FGM to withstand all the loading conditions.
2. Proper gradation is necessary for the FGM as it gives better strength and thermal gradation property. The present problem is observed to give better results for a volume fraction index ‘n=3’.
3.
4. The study has led to a material saving of 58.125% in terms of mass keeping in mind all the factors like factor of safety.Study can act as a guiding approach for optimization of other components with application in thermo-mechanical field.

VI. APPENDIX

Appendix 1

TABLE I MATERIAL PROPERTIES OF ZrO_2 AND Ti-6Al-4V[14]

Materials	***E*** (Youngs Modulus) **in GPa**	***υ*** (Poisson ratio)	***k*** (Thermal Conductivity) **in W/m °C**	***ρ*** (Density) **in kg/m^3**	***c*** (Specific Heat) **in J/kg °C**	***α*** (Thermal Expansion Cofficient)	Elastic Limit **in MPa**	Maximum Service Temperature **in K**
ZrO2	210	0.3	1.78	5680	420	9.4×10^{-6}	115-711	1248-2522
Ti–6Al–4V	115	0.289	7.955	4420	612	7.89×10^{-6}	786-910	620-690

Appendix 2

Variation of Different Properties by USDFLD Code

```
C    materials Aluminium and ZrO2, thickness 0.01m and index 3
C             Youngs Modulus
     FIELD(1)=((3-((X**2+Y**2)**0.5)*100)**3)*95000000000+115000000000
C             DensitY
     FIELD(2)=((3-((X**2+Y**2)**0.5)*100)**3)*1260+4420
C             ConductivitY
     FIELD(3)=((3-((X**2+Y**2)**0.5)*100)**3)*-6.175+7.955
C             EXpansion coeff
     FIELD(4)=((3-((X**2+Y**2)**0.5)*100)**3)*0.00000151+0.00000789
C             Specific heat
     FIELD(5)=((3-((X**2+Y**2)**0.5)*100)**3)*-192+612
```

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