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Design of Decentralized Controller Using RNGA for a Coupled Tank Process

M. Senthilkumar[1] and T. S. Murugesh[2]
[1&2]Assistant Professor, [1&2]Department of Electronics and Instrumentation Engineering,
Faculty of Engineering and Technology, Annamalai University, Annamalai Nagar, Tamil Nadu, India
E-Mail: senthilkumar.au@gmail.com, tsmurugesh@yahoo.com

Abstract - **This paper proposes a new analytical method for the design of decentralized Proportional-Integral (PI) controller for coupled tank process. The proposed design methodology aims to achieve the minimum interaction among the loops for Multi-Input Multi-Output (MIMO) process of first order with time delay. The decentralized controller is designed in terms of equivalent transfer function results from the pairing of MIMO process using Relative Normalized Gain Array (RNGA). The effectiveness of the proposed controller is established through simulation and real time studies. The results demonstrate that the proposed design of decentralized controller gives less interaction response coupled with good robustness.**
Keywords: **Decentralized Controller, Simple Internal Model Controller, Relative Normalized Gain Array, Coupled Tank**

I. INTRODUCTION

The decentralized control scheme sometimes called as multiloop control scheme is one of the most common schemes used for control of MIMO plants in chemical and process industries. The main reason for this is it has many practical advantages such as a simple structure, fewer tuning parameters, robustness against sensor or actuator failures, and easy understanding. Hence, many decentralized tuning methods have been reported to tune the controller in the field of process control [1]. In spite of all the methods available, designing a MIMO controller for a large scale MIMO process is a troublesome task which may not necessarily propose good control performance due to the complexity of the controller design procedures involved. In this case, the design method of decentralized controller is reduced to design of single input single output (SISO) controllers by considering the effect of other inputs of the MIMO process as disturbance. Therefore, selection of good pairs is important to reduce this kind of disturbance [2]. There are many pairing methods available for MIMO system [3]. Relative gain array (RGA) [4] is the first and most popular and powerful tool in pairing. Each element of RGA matrix is a measure of the relation between the process gain of that pair and interactions from other pairs. But RGA is not sensitive to time constant and delay of the process. So later on, dynamic RGA (DRGA) was used for pairing, which used transfer function model at all frequencies instead of steady state gain matrix [5]. Desiring to keep the simplicity of RGA and to use dynamic information of process model a new concept called effective relative gain array (ERGA) was proposed [6]. The ERGA method was later enhanced as effective relative energy array (EREA) [7]. Since the calculation of ERGA and EREA, to a great extent, relies on the critical frequency of the transfer function of each loop, two ways which define the critical frequency will generate different control structure configurations. The new pairing method namely relative normalized gain array (RNGA) was proposed to provide a less calculating and optimal pairing decision in practical applications, which describes the effects of process information in a more intuitional and comprehensive way [8]. However, RNGA loop pairing criterion proposed was limited to multivariable systems under step reference input, which makes RNGA based control configuration only suitable for industrial processes under step inputs [9]. However, other set-point changes appear even more often than step changes in industrial practice. Therefore, it is important to put forward a general loop pairing technique available to multivariable systems for various reference inputs, in order to avoid adverse effects caused by abrupt step changes.

Normalized RGA (RNGA) is the combination of the original RGA matrix and its selection rules. Using RNGA, it is possible to pair adaptively the inputs and outputs in a nonlinear and/or time variable process, where the optimal pairing may change from time to time [10]. The systematic approach to design decentralized controllers for MIMO processes containing time delay by extending the concept of relative normalized gain array (RNGA) through proper factorization of transfer function is proposed in this paper. RGA-Niederlinski index (NI)-RNGA criterion is proposed to determine the input/output paring which minimizes the cross loop interactions [11, 12]. By using the information conveyed in RNGA and RGA, an Equivalent Transfer Function (EQTF) is derived for each selected input-output pair when other loops are closed. These EQTFs have properly taken the loop interactions into account such that a given MIMO process can be perceived to be decomposed into a set of SISO processes with their transfer functions represented by EQTFs. Furthermore, the EQTFs are modified so that the control system integrity can be maintained. Finally, PI control is tuned for each loop based on the modified EQTFs. The proposed decentralized control design thus follows a systematic approach and is easy to be understood and implemented by field engineers [13].

The summarized RNGA rules are as follows:

1. Try to select pairs with large RNGA;
2. If the plant should be Decentralized Integral Controllable (DIC), avoid selecting with zero values RNGA;
3. For DIC, selected pairs should satisfy Niederlinski condition.

II. INPUT-OUTPUT PAIRING USING RNGA

Consider an *n*x*n* system with a decentralized feedback control structure as shown in Fig. 1,

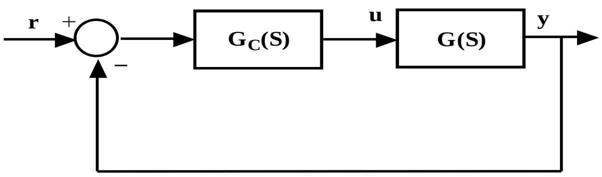

Fig. 1 Block diagram of decentralized control system

where $r = [r_1, r_2, ..., r_n]^T$, $u = [u_1, u_2, ..., u_n]^T$ and $y = [y_1, y_2, ..., y_n]^T$ are references, inputs and outputs respectively ; $G(s) = [g_{ij}]_{nXn}$ is system transfer function matrix and $G_C(s) = diag\{c_1(s), c_2(s), ..., c_n(s)\}$ is the decentralized controller; i,j = 1,2,…n are integer indices. The loop pairing problem defines the control system structure, i.e., which of the available plant inputs are to be used to control each of the plant outputs. The most popular loop pairing method is the RGA and NI based pairing rules. The relative gain for the variable pairing $y_i - u_j$ is defined as the ratio of two gains representing, first, the process gain in an isolated loop and, second, the apparent process gain in the same loop when all other loops are closed

$$\lambda_{ij} = \frac{\left(\frac{\partial y_i}{\partial u_j}\right)_{u_{l\neq j}\text{constant}}}{\left(\frac{\partial y_i}{\partial u_j}\right)_{y_{k\neq i}\text{constant}}} = g_{ij}[G^{-1}]_{ji}$$

and RGA, $\wedge(G)$, in matrix form is defined as in [6].

$$\wedge(G) = \{\lambda_{ij}, i, j = 1,2, ..., n\} = K \otimes K^{-T}$$

Furthermore, if all n loops are closed, the multi-loop system will be unstable for all possible values of controller parameters. If the NI is negative i.e., NI > 0, it provides a necessary stability condition and consequently, constitutes a complementary tool to the RGA in variable pairing selection.

III. LOOP PAIRING FOR NORMAL PROCESSES

Rewrite G(s) as G(s)= k $\otimes\bar{G}(s)$, where $\otimes$ denotes the element-by-element multiplication, k = $[k_{ij}]_{n\times n}$,: = G(0), and $\bar{G}(s) = [\bar{g}_{ij}(s)]_{n\times n}$ with $\bar{g}_{ij}(0) = 1$. Assume that the $\bar{g}_{ij}(s), \forall i, j$, is open-loop stable and its output $\bar{y}_i = \bar{g}_{ij}(s)u_j$ initially rests at zero. With u_j begin a unit step input, the average residence time (ART) is defined as

$$\tau_{ar_{ij}} = \left|\int_0^\infty (\bar{y}_i(\infty) - \bar{y}_i(t)dt)\right| \quad (1)$$

Let $T_{ar} = [\tau_{ar_{ij}}]_{nxn}$. The normalized gain matrix is defined as

$$k_N = k \odot T_{ar} \quad (2)$$

A special case is when $k_{ij}\bar{g}_{ij}(s) \equiv k_{ij}$, then $T_{ar_{ij}} = 0$. In calculating RNGA, $T_{ar_{ij}} = \varepsilon$, with $\varepsilon \to 0$ is used.

The RNGA provides reasonable information to indicate the interactions between the inputs and the outputs and is used in conjunction with RGA and NI to determine the input-output pairing. The RGA and NI are used to eliminate any structurally unstable pairing $\wedge(G) = \{\lambda_{ij}, i, j = 1,2, ..., n\} = K \otimes K^{-T}$. Given the transfer matrix G(s), the RGA (denoted by $\wedge$) and NI are defined as follows

$$\wedge = k \otimes k^{-T} \quad (3)$$

$$NI = \frac{\det k}{\prod_{i=1}^{n} k_{ij}} \quad (4)$$

In the calculation of NI, the selected input-output pairs are recorded such that their transfer functions lie on the diagonal. The RGA-NI-RNGA criterion requires that the inputs and outputs are paired [11, 12] in such a way that:

1. All paired RGA elements are positive;
2. The NI is positive;
3. The paired RNGA elements are closest to 1.0;
4. The large RNGA elements are avoided.

One of the main advantages of the above pairing rules is that the interaction evaluation depends on only the steady-state gains. This information is easily obtained from simple identification experiments or steady-state design models. A potential weakness of these rules, however, is the same fact that they only use the stead-state gains which is based on the assumption of perfect loop control to determine loop pairing.

IV. EQUIVALENT TRANSFER FUNCTION

Almost all the industry processes are open-loop stable and exhibit non-oscillatory behavior for unit-step inputs, higher-order transfer function elements can be simplified by either analytical or empirical methods to a first-order plus time delay model for interaction analysis and control system design [10]. Without loss of generality, assume all process transfer function elements, its output response in time domain to a unit step input can be described by

$$g_{ij}(s) = \frac{k_{ij}}{\tau_{ij}s+1} e^{-\theta_{ij}s} \quad (5)$$

and

$$y_i(t) = \begin{cases} 0 & t < \theta_{ij} \\ k_{ij} \times \bar{y}_i(t) & t \geq \theta_{ij} \end{cases} \quad (6)$$

respectively, where k_{ij} and $\bar{y}_i(t) = \frac{y_i(t)}{k_{ij}}$ are the steady state gain and the normalized open-loop process output, and

$$\bar{y}_i(t) = \left(1 - e^{-\frac{(t-\theta_{ij})}{\tau_{ij}}}\right) \quad (7)$$

The average residence time of loop *i-j* is given by

$$T_{ar_{ij}} = \tau_{ij} + \theta_{ij} \quad (8)$$

In control system design, two parameters are most important in describing the dynamic properties of a transfer function, i.e.,

1. Steady state gain K: the steady state gain reflects the effect of the manipulated variable u_j to the controlled variable y_i
2. Average residence time $T_{ar_{ij}}$: the average residence time is accountable for the response speed of the controlled variable y_i to manipulated variable u_j.

To measure the interaction effects, the normalized gain $K_{N,ij}$ for a particular transfer function, $g_{ij}(s)$ is given as,

$$K_{N,ij} \triangleq \frac{k_{ij}}{T_{ar_{ij}}} = \frac{k_{ij}}{\tau_{ij}+\theta_{ij}} \text{ i, j=1,2,..} \quad (9)$$

For the whole system, it can be written in a matrix form as,

$$K_N = \begin{bmatrix} K_{N,11} & K_{N,12} \\ K_{N,21} & K_{N,22} \end{bmatrix} \quad (10)$$

Similar to RGA, the Normalized relative gain matrix can be defined between output variable y_i and input variable u_j, $\wedge_{ij}$, as the ratio of two normalized gains

$$\wedge_{N,ij} = \frac{K_{N,ij}}{\widehat{K}_{N,ij}} \text{ i, j=1,2,\dots} \quad (11)$$

where $\widehat{K}_{N,ij}$ is the normalized gain between output variable y_i and input variable u_j when all other loops are closed.

The relative normalized gain array (RNGA)

$$\wedge_N = \begin{bmatrix} \wedge_{N,11} & \wedge_{N,12} \\ \wedge_{N,21} & \wedge_{N,22} \end{bmatrix} \quad (12)$$

can be calculated by

$$\wedge_N = K_N \otimes K_N^{-T} \quad (13)$$

where the operator $\otimes$ is the Hadamard product.

The relative normalized gain reflects the combined changes in both steady state gain and dynamic when all other loops are open and when all other loops are closed. To separate the two changes, first the relative average residence time γ_{ij} should be defined as the ratio of loop $y_i - u_j$ average residence time between when other loops are closed and when other are open, i.e.,

$$\gamma_{ij} \triangleq \frac{\hat{T}_{ar,ij}}{T_{ar,ij}} \text{ i, j =1,2,\dots} \quad (14)$$

Using the definition of RNGA, the equations can be written as,

$$\hat{k}_{ij} \times T_{ar_{ij}} = \frac{K_{ij} \times T_{ar_{ij}}}{\wedge_{N,ij}} \text{ i, j=1,2,.} \quad (15)$$

where $T_{ar_{ij}}$ is the average residence time of loop *i-j* when other loops are closed. Equation (15) provides both gain and average residence time change information when all other loops are closed. To separate these two changes, the definition of RGA is used here,

$$\hat{k}_{ij} = \frac{k_{ij}}{\lambda_{ij}} \text{ i, j=1,2,\dots} \quad (16)$$

When the relative average times are calculated for all the input/output combinations of the TITO process, it results in an array of the form, i.e., relative average residence time array (RARTA) which is defined as

$$\Gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \triangleq \wedge_N \odot \wedge \quad (17)$$

$$\Gamma = \begin{bmatrix} \wedge_{N,11} & \wedge_{N,12} \\ \wedge_{N,21} & \wedge_{N,22} \end{bmatrix} \odot \begin{bmatrix} \wedge_{11} & \wedge_{12} \\ \wedge_{21} & \wedge_{22} \end{bmatrix} \quad (18)$$

where the operator $\odot$ is the hadamard division.

As the relative average time is the ratio of the average residence times between when other loops are closed and when other loops are open, $\hat{T}_{ar,ij}$ represent the dynamic changes of the transfer function $g_{ij}(s)$ when other loops closed. By the definition of RARTA,

$$\hat{T}_{ar,ij} = \gamma_{ij} \times T_{ar,ij} = \gamma_{ij} \times \tau_{ij} + \gamma_{ij} \times \theta_{ij} \quad (19)$$

The average residence time of loop *i-j* th when other loops are closed is the open loop average residence time scaled by a factor γ_{ij}. In process control, steady state gain, time constant and time delay are the parameters that are of topmost interest for control system design. By using RGA and RARTA information, gain and phase changes of a transfer function element when other loops closed can be uniquely determined, i.e., a transfer function element of a MIMO process when other loops are closed can be approximated by a transfer function element having the same form as the open-loop transfer function element, but the steady state gain, time constant and time are scaled by $\frac{1}{\lambda_{ij}}$ and γ_{ij}, respectively, i.e.,

$$\hat{g}_{ij}(s) = \hat{k}_{ij} \times \frac{1}{\hat{\tau}_{ij}s+1} e^{-\hat{\theta}_{ij}s} = \frac{k_{ij}}{\lambda_{ij}} \times \frac{1}{\gamma_{ij}\tau_{ij}s+1} e^{-\gamma_{ij}\theta_{ij}s} \text{ i,j=1,2,\dots} \quad (20)$$

V. DECENTRALIZED CONTROLLER DESIGN USING EQTF

Since EQTF have incorporated the information of loop interactions, the MIMO process can be decomposed into a set of SISO processes then decentralized PID controllers can be designed to stabilize these SISO loops independently. In application, however, it is desirable that the MIMO system remains stable if any of the loops is taken in or out of service. This requires that the controllers be designed conservatively. Such motivates the use of modified EQTF which keeps the same form of the EQTF but has parameters taking the larger values of EQTF and its corresponding open-loop transfer function,

$$\tilde{g}_{ij}(s) = \frac{\tilde{k}_{ij} e^{-\tilde{\theta}_{ij}s}}{(\tilde{\tau}_{ij}s+1)} \quad (21)$$

where $\tilde{g}_{ij}(s)$ is the modified EQTF in which

$$\tilde{k}_{ij} = max\{k_{ij}, \tilde{k}_{ij}\}, \quad \tilde{\tau}_{ij} = max\{\tau_{ij}, \tilde{\tau}_{ij}\}, \quad \tilde{\theta}_{ij} = max\{\theta_{ij}, \tilde{\theta}_{ij}\} \quad (22)$$

That the larger parameters usually imply the more challenging situations for control, which sequentially implies that the controller design, will be more conservative as compared to using the smaller parameters. As each controller design becomes a SISO case, any good PID tuning methods may apply. The simple internal model control (SIMC) tuning method is adopted for simplicity and robustness. The SIMC controller settings for the first-order with time delay process [14] is given as

$$g(s) = k \frac{e^{-\theta s}}{(T_I s+1)} \quad (23)$$

$$k_c = \frac{1}{k}\frac{T_I}{\tau_c+\theta} \quad (24)$$
$$T_I = min\{\tau_1, 4(\tau_c+\theta)\} \quad (25)$$

VI. COUPLED TANK PROCESS

Coupled tank is prominently used in petro-chemical industries, paper making industries as well as in water treatment industries for processing chemicals or mixing treatment. The control of level of fluid in tanks is a challenging problem due to interactions between the tanks and also serves as a MIMO process. The schematic diagram of coupled tank process is shown in Fig.2. The controlled variables are levels of tanks. The levels of the tanks are maintained by manipulating the inflows to the tanks.

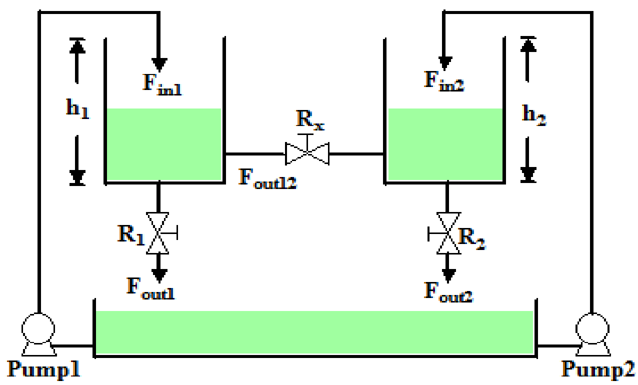

Fig. 2 Schematic diagram of coupled tank process

The mass balance equation of the process is

$$A_1\frac{dh_1}{dt} = k_{pp1}u_1 - \beta_1 a_1\sqrt{2gh_1} - \beta_2 a_{12}\sqrt{2g(h_1-h_2)} \quad (26)$$
$$A_2\frac{dh_2}{dt} = k_{pp2}u_2 - \beta_x a_{12}\sqrt{2g(h_1-h_2)} - \beta_2 a_2\sqrt{2gh_2} \quad (27)$$

The identified transfer function of coupled tank process [15] is

$$G(s) = \begin{bmatrix} \frac{16.66e^{-12.89s}}{214.03s+1} & \frac{6.69e^{-72.57s}}{204.93s+1} \\ \frac{9.23e^{-35.01s}}{256.49s+1} & \frac{11.38e^{-25.04s}}{169.15s+1} \end{bmatrix}$$

The Equivalent transfer function and the controller settings using Simplified Internal Model Control (SIMC) method [14] are obtained using the identified model and presented in Table I

TABLE I SIMC SETTINGS

$\hat{g}_{ii}(s)$	K_c	T_I
$\hat{g}_{11}(s) = \frac{11.5641e^{-10.6329s}}{176.65s+1}$	0.1729	42.5316
$\hat{g}_{22}(s) = \frac{7.7457e^{-20.65549s}}{139.53s+1}$	0.25819	82.6219

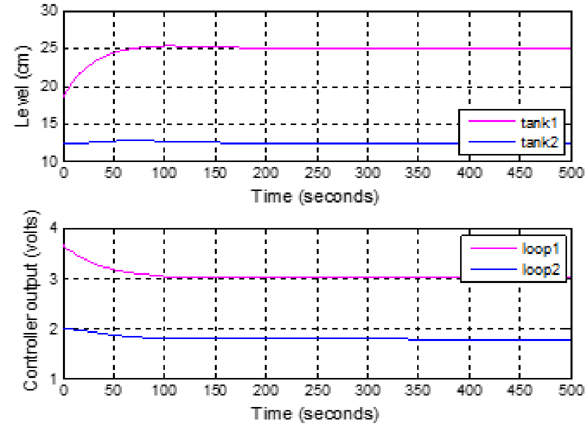

Fig. 3 Closed loop response of coupled tank process for set point Change in tank 1

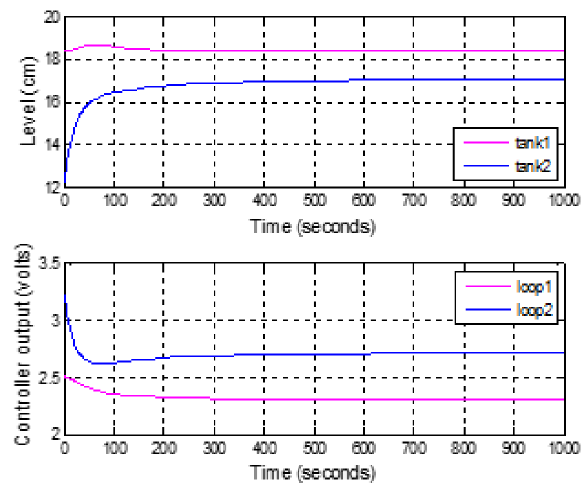

Fig. 4 Closed loop response of coupled tank process for set point change in tank 2

The decentralized controller designed is implemented through MATLAB software and the closed loop response for the coupled tank process for a set-point change in tank1 from its operating value of 18.32cms to 25cms is shown in Fig.3. Similarly the closed loop servo response for set-point change in tank 2 to 17cms from its operating value of 12.23cms and its controller output are shown in Fig.4.

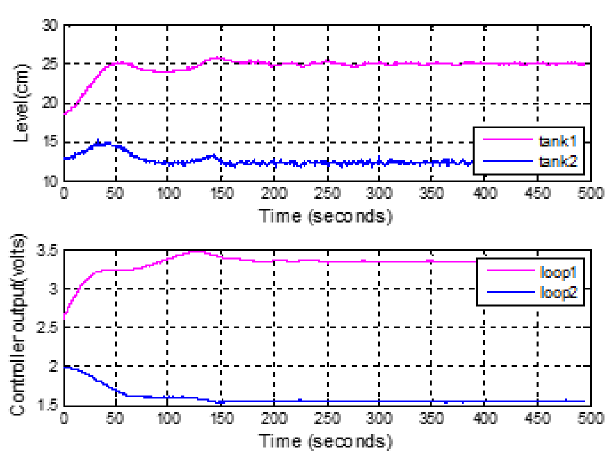

Fig. 5 Closed loop real time response of coupled tank process for Set point change in tank1

The closed loop real time response for coupled tank process for set point change in tank1 and tank 2 are depicted in Fig.5 and Fig.6 respectively. The controller performance is measured and evaluated which are shown in Table II. The stability of the system is checked by plotting the characteristic loci plot and shown in Fig.7. From the characteristic loci plot it is observed that the plot has not encircled the point -1+j0, so that the system is stable.

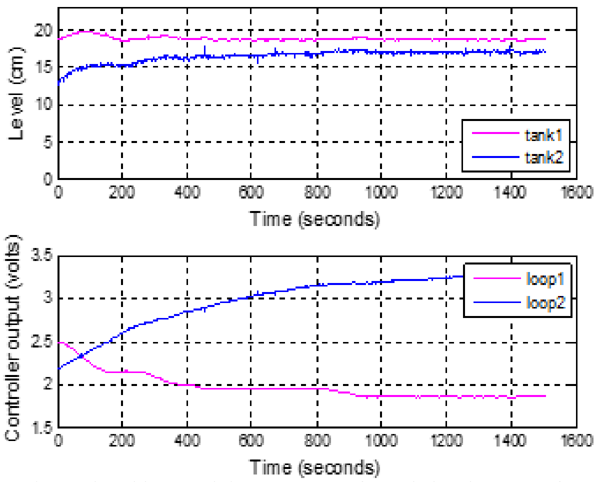

Fig. 6 Closed loop real time response of coupled tank process for Set point change in tank 2

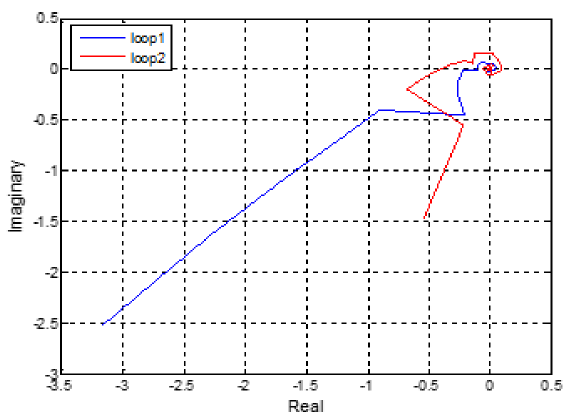

Fig. 7 Characteristic loci plot

TABLE II PERFORMANCE MEASURES

Set point change	Loop 1		Loop 2	
	ISE	IAE	ISE	IAE
Tank 1	549.07	164.48	18.83	17.19
Tank 2	10.10	48.32	357.77	225.87

VII. CONCLUSION

In this paper, loop pairing method is analyzed for interaction measurement of coupled tank MIMO system. In pairing method both the transient and steady state information of the process are considered to find the interactions between the loops. Based on RGA-NI-RNGA criterion the pairing is achieved for coupled tank process then the Equivalent transfer function is derived for the selected input-output pairs using RNGA, thereby the MIMO process being decomposed into SISO process. Hence the decentralized controllers are designed independently for the coupled tank MIMO process as SISO process. The unique advantage of the proposed approach is its simplicity in carrying out a systematic decentralized control design, which is easier to be understood and implemented by field engineers.

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