Asian Journal of Electrical Sciences
ISSN: 2249-6297 (P) Vol.6 No.1, 2017, pp.23-28
© The Research Publication, www.trp.org.in
DOI: https://doi.org/10.51983/ajes-2017.6.1.1992

Sallen-Key Topology, MFB and Butterworthy in Bandpass Design for Audio Circuit Design

Oyebola Blessed Olalekan[1] and Odueso Victor Toluwani[2]
[1]Gateway (ICT) Polytechnic Saapade
[2]Department of Electrical/Electronic Engineering
University of Ibadan
Email: blessedolalekan@gmail.com,vodueso797@stu.ui.edu
(Received 15 March 2017; Revised 9 April 2017; Accepted 24 April 2017; Available online 30 April 2017)

***Abstract* - This paper succinctly studied active filter topologies: Sallen-key circuit, MFB and Butterworth in band pass design for audio circuits. It has been able to identify distinctive suitable features with applications in audio frequencies range. The work also serves as a foundational study for an advanced work on active audio filters. Sallen-Key pluses with quality factor (Q) that can be varied via the inner gain (G) without modifying the mid frequency (f_m). Notably, the Q and A_m cannot be adjusted independently. Butterworth, maximally flat magnitude response in the pass band, has monotonic amplitude response with an excellent flat pass band, less phase shift, and better transient results.**
***Keywords:* audio, circuit, design, filter, opamp, topology.**

I. INTRODUCTION

The Sallen–Key topology is an electronic filter topology used to implement second-order active filters that is particularly valued for its simplicity. It is a degenerate form of a voltage-controlled voltage-source (VCVS) filter topology. A VCVS filter uses a unity-gain voltage amplifier with practically infinite input impedance and zero output impedance to implement a 2-pole low-pass, high-pass, band pass, band stop, or all pass response. The unity-gain amplifier allows very high Q factor and pass band gain without the use of inductors (1). A Sallen–Key filter is a variation on a VCVS filter that uses a unity-gain amplifier (i.e., a pure buffer amplifier with 0 dB gain). Because of its high input impedance and easily selectable gain, an operational amplifier in a conventional non-inverting configuration is often used in VCVS implementations.
Implementations of Sallen–Key filters often use an operational amplifier configured as a voltage follower; however, emitter or source followers are other common choices for the buffer amplifier. VCVS filters are relatively resilient to component tolerance, but obtaining high Q factor may require extreme component value spread or high amplifier gain (2). Higher-order filters can be obtained by cascading two or more stages.

The general Sallen-key topology, in Fig.1, allows for separate gain setting via $A_o = 1+(R_4/R_3)$. However, the unity-gain topology in Fig. 2 is usually applied in filter designs with gain accuracy, unity gain, and low Qs (Q<3).

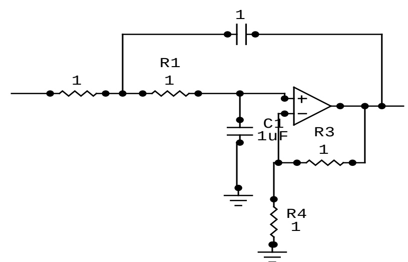

Fig.1 General sallen –key low-pass filter

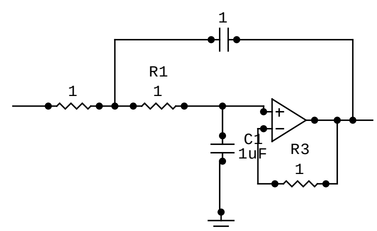

Fig. 2 Unity gain Sallen-key low pass filter (1)

The transfer function of the circuit in Fig.1 is

$$A_{(s)} = \frac{Ao}{1+wc[c1(R1+R2)+(1-Ao)R1C2]s+wc2R1R2C1C2S2}$$

For the unity gain circuit in Fig. 2 (A_o =1), the transfer function simplifies to:

$$A_{(s)} = \frac{1}{1+\omega cC1(R1+R2)S+\omega c2R1+R1R2C1C2s2}$$

The coefficient comparison between this transfer function yields:

$$A_o = 1$$
$$a_1 = wcC_1(R_1+R_2)$$
$$b_i = wc^2R_1R_2C_1C_2$$

Given C_1 and C_2, The resistor valves for $R_1$and R_2 are calculated through:

$$R_{1.2} = a_1c_2 = \frac{\sqrt{a1c2}\text{-}4b_1C_1}{4\pi fcC1C2}$$

In order to obtain real valves under the squre root must satisfy the following conditions

$$C_2 \geq \frac{C_1 \mathbf{4b1}}{a_1^{\,2}}$$

A special case of the general Sallen key topology is the application of equal resistor valves and equal capacitor valves: $R_1=R_2=R$ and $C_1=C_2=C$. the general transfer function (3) changes to:

$$A(s) = \frac{Ao}{1+wcRc(3s-Ao)s+(WcRC)22} \qquad \text{with} \quad A_0 = 1+\frac{R4}{R3}$$

The coefficient comparison also yields:

$$a_1 = \boldsymbol{\omega}cRC(3\text{-}Ao)$$
$$b_1 = (\boldsymbol{\omega}cRC)^2$$

Given C and solving for R and Ao results in:

$$R = \sqrt{\frac{b1}{2\pi fc}} \qquad \text{and } A_o = 3-\frac{a1}{\sqrt{b1}} = 3-\frac{1}{Q}$$

Thus, A_o depends solely on the pole quality Q and vice versa; Q, and with it the filter type, is determined by the gain setting of A_o:

$$Q = \frac{1}{3-Ao}$$

The circuit in Fig.3 allows the filter type to be changed through the various resistor ratios $R_4/R_{3.}$

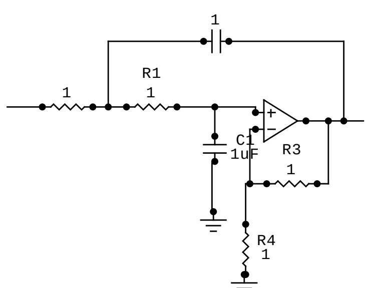

Fig.3 Adjustable second order low pass filter

Table 1 list the coefficients of the second order filter for each filter type and gives the resistor ratios that adjust the Q.

TABLE 1 SECONDS-ORDER FILTER COEFFICIENTS

Second order	Bessel	Butterworth	3-dB Tschebyscheff
a_1	1.3617	1.4142	1.005
b_1	0.618	1	1.9305
Q	0.58	0.71	1.3
R_4/R_3	0.288	0.588	0.234

Source ([14])

A. *Multiple Feedback Topologies*

The MFB, Fig 4, topology is commonly used in filters that have high Qs and require a high gain.

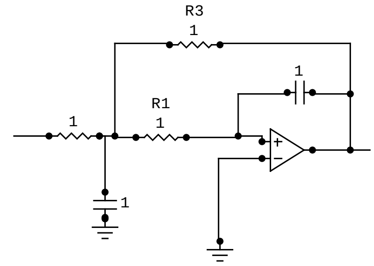

Fig.4 Second-order MFB low pass filter

1.Higher Order Low-pass Filters

Higher order low pas filter are required to sharpen a desired filter characteristic (4). For that purpose, first order and second order filter stages are connected in series, so that the product of the individual frequency responses results in the optimized frequency response of the overall filter. In order to simply the design of the partial filters, the coefficients a_1 and b_1 for each filter are listed in the coefficient Tables.

2.Second Order Filter

Fig. 5 illustrated second-order unity-Gain Sallen-Key Low-Pass Filter.

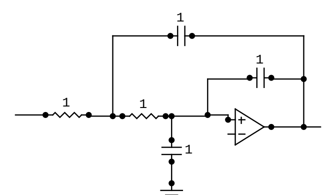

Fig.5 Second-order unity-gain sallen-key low-pass filter

$$C_2 \geq C_1 \frac{4b_2}{{a_2}^2} \;\; ; \quad R_1 = \frac{a_2C_2 - \sqrt{{a_2}^2 - 4b_2C_1C_2}}{4\pi f_c C_1 C_2}$$
$$And \quad R_2 = \frac{a_2C_2 + \sqrt{{a_2}^2 - 4b_2C_1C_2}}{4\pi f_c C_1 C_2}$$

3.Third Order Filter

The calculation of the third filter is identical to the calculation of the second filter, except that a_2 and b_2 are replaced by a_3 and b_3, thus resulting in different capacitor and resistor valves. Fig. 6 shows the final filter circuit with its partial filter stages.

$$C_2 \geq C_1 \frac{4b3}{a32}$$

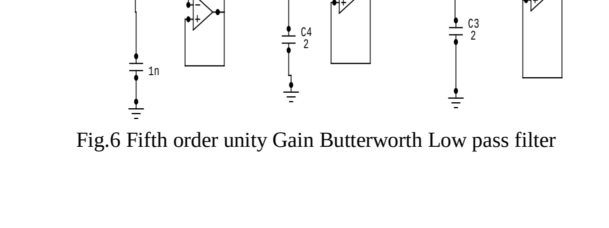

Fig.6 Fifth order unity Gain Butterworth Low pass filter

B. High Pass Filter Design

By replacing the resistor of the low pass filter with capacitors, and its capacitors with resistors, a high pass filter is created, see Fig. 7.

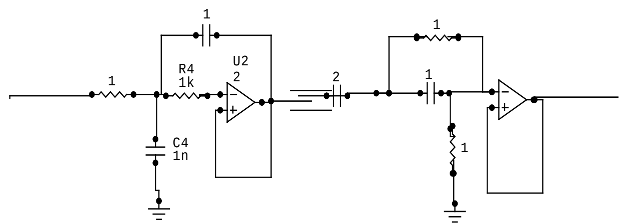

Fig.7 Low Pass Filter Transition through Component Exchange

The general transfer function of the high pass filter is:

$$A_{(s)} = \frac{A}{\pi(1+\frac{ai}{s}+\frac{bi}{s}}$$

With $^{A}\omega$ being the pass-band gain, filter coefficient could be determined from the Table in appendix A.

1.Multiple Feedback Architecture

The MFB topology is commonly used in filters that have Qs and require a high gain. To simply the computation of the circuit, capacitors C_1 and C_2 assume the same valve (C_1= C_3 = C) (5). The transfer function of the circuit is

$$A(S) = -\frac{\frac{C}{C2}}{1+\frac{2C+C2}{\omega CR1CC2}\cdot\frac{1}{s}+\frac{2C+C2}{wcR1CC2}\cdot\frac{1}{S}2}$$

Through coefficient comparison with equation above, obtain the following relations:

$$A\omega = \frac{C}{C2}$$

$$a_1 = \frac{2C+C2}{\omega cR1CC2} \qquad R_1 = \frac{1-2A\alpha}{2\pi f.C.a1}$$

$$b_1 = \frac{2C+C2}{\omega cR1CC2} \qquad R_2 = \frac{a1}{2fc.b1C2(1-2A\alpha)}$$

Given capacitors C and C2, are solving for resistors R1 and R2. The pass-band gain A (∞) of a MFB high pass filter can very significantly due to the wide tolerances of the two capacitors C and C2. To keep the gain variation at a minimum, it is necessary to use capacitors with tolerance valves.

II . BAND-PASS FILTER DESIGN

In section the previous section, replacing the term S in the low pass generated a high pass response transfer function with the transformation I/S. likewise, a band pass characteristics is generated by replacing the S term with the transformation (6)

$$\frac{1}{\Delta\Omega}\left(s+\frac{1}{s}\right)$$

In this case, the pass band characteristics of a low pass filter is transformed into the upper passband half of a band filters. The upper pasband in then mirrored at the mid frequency, f_m=1, into the lower passband half. The corner frequency of the low pass filter transforms to the lower and upper 3dB frequencies is defined as the normalized bandwidth (ΔΩ) (7).

$$\Delta\Omega = \Omega 2 - \Omega 1$$

The normalized mid frequency, where Q=1, is:

$$\Omega_m = 1\Omega_2\Omega_1$$

In analogy to the resonant circuit, the quality factor Q is defined as the ratio of the mid frequency (f_m) to bandwidth

(B): $$Q = \frac{fm}{B} = \frac{fm}{f2-f1} = \frac{1}{\Omega 2-\Omega 1} = \frac{1}{\Delta\Omega}$$

The simplest design of a band pass filter is the connection of a high pass filter and a low pass filter in series, which is commonly done in wide band filter application. Thus, a first order high pass and first order low pass provide a second order band pass, while a second order high pass and a second order low pass result in a fourth order ban pass response. In comparison to wide band filters, narrow band filters of higher order consist of cascade second order band pass filter that use the Sallen key or the multiple feedback (MFB) topology (8).

C. Second Order Band Pass Filter

To develop the frequency response of a second order band pass filter, apply transformation to a first order low pass transfer function:

$$A_{(s)} = \frac{Ao}{1+s} \quad ; \quad \text{Replacing } \frac{1}{\Delta\Omega}\left(s+\frac{1}{s}\right)$$

Yield the general transfer function for a second order band pass filter:

$$A(s) = \frac{Ao\Delta\Omega s}{1+\Delta\Omega s+s}$$

When designing band pass filter, the parameters of interest are the gain at the mid frequency(A_m) and the quality factor (Q), which represents the selectivity of a band pass filter. Therefore, replace Ao with A_m and Ω with I/Q gives (9):

$$A(s) = \frac{Am}{Q}S$$

Fig.8 shows the normalized gain response of a second order band pass filter for different Qs.

1.Sallen Key Topology

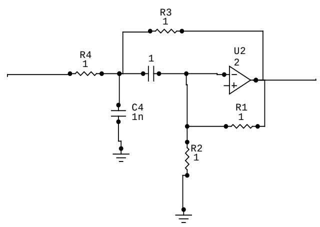

Fig.8 Sallen-key Band-Pass

The Sallen-key band-pass circuit in Fig.8 has the following transfer function:

$$A(s) = \frac{G.RCwms}{1+RC\omega m(3-G).s+RC\omega ms}$$

Through coefficient comparison obtained the following equation:

$$\text{Mid frequency: } f_m = \frac{1}{2\pi RC}$$
$$\text{Inner gain: } G = 1+\frac{R2}{R1}$$
$$\text{Gain at } f_m\text{: } A_m = \frac{G}{3-G}$$
$$\text{Filter quality: } Q = \frac{1}{3-G}$$

The Sallen-key circuit has the advantages that quality factor (Q) can be varied via the inner gain (G) without modifying the mid frequency (f_m). A drawback is, however, that Q and A_m cannot be adjusted independently; care must be taken when G approaches the valve of three, because then A_m becomes infinite and causes the circuit to oscillate. To set the mid frequency of the band pass, specify f_m and C and the R is solved for

$$R = \frac{1}{2\pi fmC}$$

Because of the dependency between Q and Am, there are two options to solve for R_2: either to set the gain at mid frequency:

$R_2 = \frac{2Am-1}{1+Am}$; $R_2 = \frac{2Q-1}{Q}$ or to design for a specified Q.

III. MULTIPLE FEEDBACK TOPOLOGIES

Multiple feedback topologies are an electronic filter topology which is used to implement an electronic filter by adding two poles to the transfer function (10). A diagram of the circuit topology for a second order low pass filter is shown in the Fig. 9.

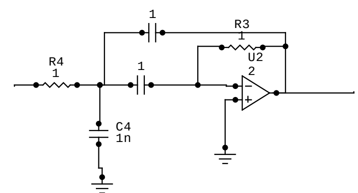

Fig.9 MFB band pass

The MFB band pass circuit in Fig. 9 has the following transfer function:

$$A(s) = \frac{-\frac{R_2R_3}{R_1+R_3}C_{\omega_m}.S}{1+\frac{2R_1R_3}{R_1+R_3}C_{\omega_m}.S+\frac{R_1R_2R_3}{R_1+R_3}C^2.\omega_m{}^2.S^2}$$

The MFB band pass allows adjusting Q, A_m, and f_m independently. Bandwidth and gain factor do not depend on R_3. Therefore, R_3 can be used to modify the mid frequency without affecting bandwidth, B, or gain A_m. for low values of Q, the filter can work without R_3, however, Q then depends on A_m via: $A_m = 2Q^2$. The schematic diagrams in Fig.10, Fig.11, Fig. 12 and Fig. 13 show Passive Twin T Filter, Active Twin T Filter, Passive Wien Robinson Bridge and Active Wien Robinson Filter respectively.

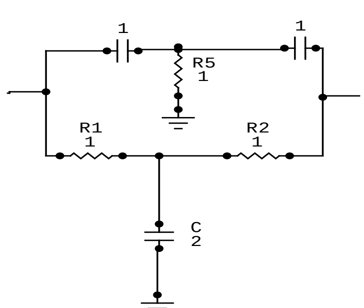

Fig.10 Passive Twin T Filter

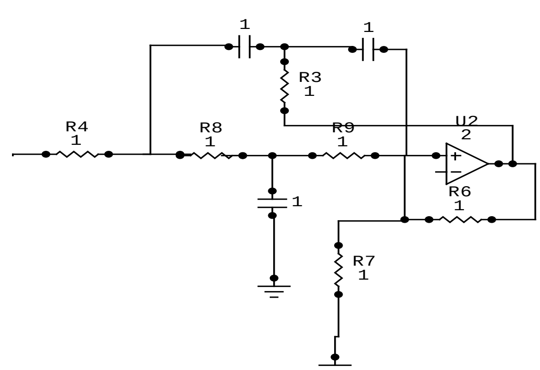

Fig.11 Active Twin T Filter

The transfer function of the active twin T filter is:

$$A(S) = \frac{K(1+S2)}{1+2(2-K).S+S2}$$

Mid –frequency: $f_m = \frac{1}{2\pi RC}$

Inner gain: $G=1+\frac{R2}{R1}$

Passband gain: Ao= G

Rejection quality Q: $\frac{1}{2"(2-G)}$

D. Active Wien Bobinson filter

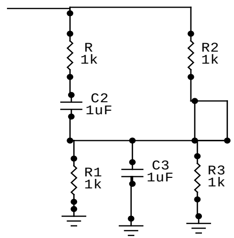

Fig.12 Passive Wien Robinson Bridge

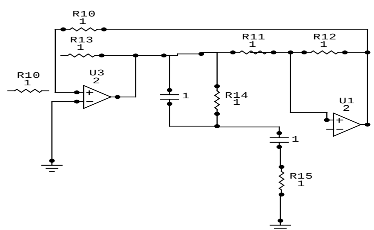

Fig.13 Active Wien Robinson Filter

The active Wien Robinson filter has the transfer function:

$$A(S) = -\frac{\beta}{1+\alpha}(1+S^2)$$

$$1+\frac{3}{1+\alpha}.S+S^2$$

With $\alpha = \frac{R2}{R3}$ and $\beta = \frac{R2}{R4}$

And with comparing the variables, it provides the equations that determines the filter parameter

Mid frequency: $f_m = \frac{1}{2\pi RC}$

Passband gain: $Ao = \frac{\beta}{1+\alpha}$

Rejection quality: $Q = \frac{1+\alpha}{3}$

To calculate the individual component values, the following design procedures are established:

- **i.** define f_m and C and calculate R with: $R=\frac{1}{2\pi fmc}$
- **ii.** specify Q and determine α via: $\alpha = 3Q-1$
- **iii.** Specify Ao and determine β via: β=-Ao.3Q
- **iv.** Define R_2 and calculate R_3 and R_4 with: $R_3=\frac{R2}{\alpha}$ and $R_4=\frac{R2}{\beta}$

1.Pass Filter Design

The general transfer function of an all pass is then

$$A(S) = \pi(1-a_i s+b_i s^2)$$
$$\pi(1+a_i s+b_i s^2)$$

with a_i and b_i being the coefficients of a partial filter

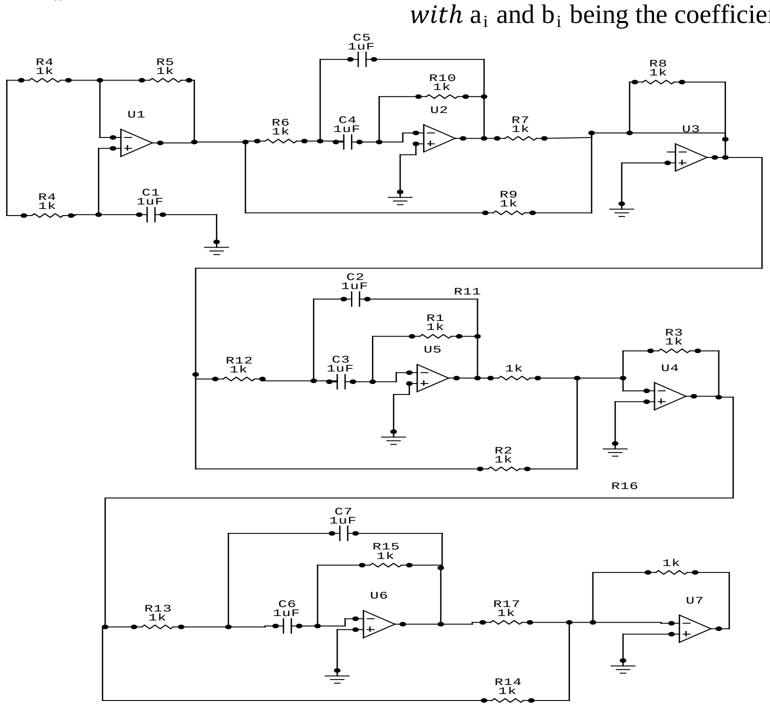

Fig. 14 Seven-Order All-Pass Filter (11)

Another type of topology called state- variable (Bohn 1976), band pass circuits have the best performance for the cost is no object designs; other filter such as transverse filter (which main advantage that it offers linear phase) and shelving filter had all been used for similar projects. But due economic reasons MFB is opted for band allocation (11).

2.Tone Control (BaxandalI) Circuit

This gives independent variation of bass and tremble without switches; it is fully symmetrical, and there is no interaction between the controls. The OP-AMP acts as a buffer. Frequency response is absolutely flat. Its bass is defined at frequency less then resonant frequency is that where the capacitance and inductance is equal. This provides to give more selectively of desired tone. Tone control of audio systems involves altering the flat response in order to attain more low frequencies or more high ones, dependent upon listeners' preference. The circuit produces 20Db (4^{th} order) of bass or treble boost or cut as set by the variable resistance. The response of the circuit is as shown. Baxandall Tone controls: when centered, there is neither loss nor gain, and the op-amp acts as a buffer.

3.Crossover

It is hard to make a loud speaker that is capable of handling the entire audio spectrum more difficult to make one that does this well hence the needs for crossover. The use of higher other filter allows loudspeakers to be played at the limits of their efficiency. Higher filter can also be beneficial in compensating for natural peaks within the listening environments (vehicles).

Crossover is an essential part of an audio system (sometime ignored); it splits frequency so that each speaker receives a certain range of frequencies to avoid speaker damage and to maintain overall balance. Speakers are designed only to efficiently play in range of frequencies, if the frequencies are played than the speaker will produce distortion, which will eventually destroy it. If a system with subwoofers (20 to 100Hz) and full ranges speakers does not have crossover, then the subs will be playing while the full rage speakers will be playing from 60Hz. There is an "overlap" of frequencies between 50 and 100Hz, yielding to a non-balanced system. These are three types of crossover: high pass, low pass and band pass. The crossover cut does not block undesired frequencies completely unless it is digitalized) frequencies progressively (12). The minimum blocking should be 6dB/ octave (for the least filter order) that is the level at the speaker.

4.Q-Choice

Most audio applications require a maximum Q of about 4.0 is suited for $_{1/3}$ octave filter set Q of 10 is too high to be useful in most audio application. Bandwidth is measured at the -3dB frequencies on either sides of the resonant peak (ωo). $\omega a \times \omega_{c2:}$ increasing Q does nothing to the roll off slope.

IV. CONCLUSION

The Sallen-key circuit has the advantages that quality factor (Q) can be varied via the inner gain (G) without modifying the mid frequency (f_m). A drawback is, however, that Q and A m cannot be adjusted independently. Butterworth has monotonic amplitude response with a maximally flat pass band, less phase shift, and better transient results; in conclusion, it is the preferred choice (in this project it was only be used in cross over design which was not constructed due to time and financial constrain). Advantages maximally flat magnitude response in the pass band. Good all round performance plus response better than Chebyshev. Rate of attenuation is better than Bessel.

REFERENCES

[1] Wikipedia. [Online] February 16, 2017. [Cited: March 11, 2017.] https://en.wikipedia.org/wiki/Sallen%E2%80%93Key_topology.

[2] *OMICS Publishing Group.* [Online] 2014. http://research.omicsgroup.org/index.php/Sallen%E2%80%93Key_topology.

[3] Thomas, Kugelstadt. *OpAmps - The University of Sydney.* [Online]

[4] Darren, Ashby, et al., et al. Circuit Design: Know It All. 2011, pp. 704-730.

[5] Lessons In Electric Circuits -- Volume III (Semiconductors) - Chapter 3. [Online] 2017. [Cited: March 2, 2017.] https://www.ibiblio.org/kuphaldt/electricCircuits/Semi/SEMI_3.html.

[6] Bruce, Carter. Op Amps for Everyone. [Online] 2003. https://books.google.com.ng/books?id=JVOhdFHgVFUC&pg=PA287&lpg=PA287&dq=previous+section,+replacing+the+term+S+in+the+low+pass+generated+a+high+pass+response+transfer+function+with+the+transformation+I/S.+likewise,+a+band+pass+characteristics+is+generated.

[7] *Researchgate.* [Online] February 7, 2012. [Cited: jANUARY 6, 2017.] https://www.researchgate.net/.../Please_tell_me_Why_the_cutoff..

[8] Kugelstadt, Thomas. Active Filter Design Techniques. *Texas Instruments.* [Online] 2008. [Cited: March 9, 2017.] https://focus.ti.com/lit/ml/sloa088/sloa088.pdf.

[9] Active Band Pass Filter - Electronics Tutorials. [Online] 2016. [Cited: May 4, 2016.] http://www.electronics-tutorials.ws/filter/filter_7.html.

[10] Electronic filter topology. *Wikipedia.* [Online] November 9, 2016. https://en.wikipedia.org/wiki/Electronic_filter_topology.

[11] Analog Filter Design - FDI Online Design Guide - Frequency Devices. [Online] August 2003. [Cited: March 26, 2017.] http://www.freqdev.com/guide/analog.html.

[12] Kirkland, E. J and Littauer, R. laboratory manual - CLASSE Cornell. [Online] July 28, 2010. [Cited: March 26, 2017.] https://www.classe.cornell.edu/~ib38/teaching/p360/lab_manual_20100728.pdf.