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Multi-Objective, Short-Term Hydrothermal Scheduling Based on Weighting Method Using Particle Swarm Optimization

Abraham George[1], Bhagwan Shree Ram[2] and P. Premalatha[3]
Department of Electrical & Electronics Engineering, MVJ College of Engineering, Bangalore, Karnataka, India
E-Mail: george.ab2008@gmail.com, bhagwan62@yahoo.com, premalatha_giri@rediffmail.com
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Abstract **- This paper describes an algorithm to solve multi-objective, short term hydrothermal scheduling problem incorporating a few new techniques to make it more simple and fast. Multi-objective hydrothermal scheduling problem allocates the system generation among the available hydro and thermal generators such that an overall satisfactory solution to the problem of optimizing the objectives viz. cost of generation, emission of NOx, emission of CO2 and emission of SO2 is obtained. There are several methods to solve this problem but 'Weighting method' stands out of all. In weighting method, a composite objective function, combining all the given objectives, is formed assigning a suitable weight to each objective provided that the sum of the weights is equal to one. Normal practice is to assume a set of weight vectors, solve the problem considering each weight vector and go for the best overall satisfactory solution. In the method described in this paper,a set of weight vectors are randomly generated and the problem is solved for each weight vector. Further the weight vectors are modified and the problem is solved for each modified weight vector. This process is continued till the index of satisfaction is the highest. Modification of the weight vectors is done incorporating a new technique called 'search space reduction technique'and introducing fuzzy acceleration factors.**

With modification of weight vectors using PSO highly efficient weight vectors/best possible solution to the hydrothermal scheduling problem can be obtained in minimum number of trials. The algorithm developed is tested on different systems and satisfactory solutions are obtained. The algorithm is simple and fast and gives a set of solutions with index of satisfaction falling in a narrow range for the multi-objective, short term hydrothermal scheduling problem.

Keywords: **Particle Swarm Optimization, search space reduction technique, hydrothermal scheduling, fuzzy logic**

1. INTRODUCTION

The classical hydrothermal scheduling problem had only cost objective till the recent past. But, as emission levels kept on increasing, public became more aware of environmental protection. Hence environmental objectives were added to cost objective resulting in a multi-objective problem. Multi-objective hydro thermal scheduling problem emphasizes on reduction of cost of operation of thermal units and emission of hazardous pollutants.

Lately, many approaches have been suggested to solve hydrothermal scheduling problem.

In the paper by Huifeng Zhang *et.al.* [1] three chaotic sequences based multi-objective differential evolution (CS-MODE) is proposed to solve theshort term hydrothermal optimal scheduling with economic emission problem, and it utilizes elitist archive mechanism to retain the non-dominated individuals, which improves the convergence ability in the differential evolution, and a heuristic two-step constraint-handling technique is utilized to handle those complex equality and inequality constraints in the scheduling problem.

Abdollah Ahmadi *et.al.* proposed amethod [2] which solves the multi-objective, short term hydrothermal scheduling problem using lexicographic optimization and Normal Boundary Intersection method. The main positive point with this approach is that it avoids the selection of arbitrary parameters and produces a set of evenly distributed points regardless of the objectives' scales. Afterwards, the most preferred solution among all Paretosolutions is selected utilizing a fuzzy satisfying method.

Sayeed Salam[3] developed an algorithm with economy as an objective and SO_2 emission as a constraint, de-committing highly polluting units during certain sub-intervals to reduce the impact on environment.

A new heuristic search technique based on binary successive approximation using stochastic models was proposed by J.S. Dhillon et.al. [4]. They considered five objectives; economy, emission of NOx, emission of SO2, emission of CO2 and variance of power.

M Basu *et. al.* developed an algorithm [5] for economic-emission load dispatch in multi-objective environment, using simulated annealing on interactive fuzzy satisfying method.

Abdullah Konak and David Coit, in their paper on multi-objective optimization [6], states the conflicting nature of different objectives in a real life problem. On such an occasion, a set of non-inferior solutions satisfying each objective to an acceptable level, is to be found and the best overall satisfactory solution is to be identified.

A model incorporating hydrothermal co-ordination, unit commitment and economic load dispatch was proposed by Esteban Gil et. al. [7]. They decomposed the hydrothermal

scheduling. problem into the above mentioned sub problems. The model was implemented using Genetic Algorithm. Three different types of crossover, two different types of mutation and two repair operators were used in the algorithm to improve the solution efficiency.
J S Dhillon, S C Parti and D P Kothari suggested a method [8] in which fuzzy decision-making methodology is exploited to decide the generation schedule of a short range, fixed head hydrothermal problem.

A George and D P Kothari proposed a simple algorithm [9] in which a random search method is used to for solving multi-objective, short term hydrothermal scheduling problem. The algorithm was implemented using Genetic Algorithm.

Emission levels in many counties are dangerously high, leading to environmental hazards. Utilities are often forced to modify their operational strategies to minimize pollution. Also the available energy sources are to be effectively scheduled for efficient operation of a power system keeping economy in view. Hence an effective hydrothermal scheduling is essential as hydrothermal energy sources constitute the major portion of the available energy sources in most of the countries.

The hydrothermal scheduling problem is a non-linear optimization problem with equality and inequality constraints. PSO based search is normally considered faster in locating the global optimum when search space is large, noisy and multi modal.

There are two general approaches to multi-objective optimization. One approach is to combine various objective functions into a single composite objective function and the other is to consider one objective as the prime one and the remaining as constraints. Difficulty in accurately determining the weight is the drawback of the first approach while the second approach has the drawback of establishing constraining values, which can be rather arbitrary. In this study the first approach is used. In most of the real life problems, objectives are conflicting in nature. In such cases, optimization with respect to a single objective yields results which are unacceptable to other objectives. And hence simultaneously optimizing all the objectives is impossible. A way to solve the multi-objective problem is to find a set of solutions, each of which satisfies all the objectives to a certain degree, without being dominated by any other solution.

A technique called 'Search space reduction technique', is incorporated in this study. This technique can be applied to a variety of single variable algebraic equations of higher order to locate the real roots of the equation by repeated search. The gradient of the function to be optimized is equated to zero and the real roots of the resulting equation are found by exploring the search space several times. Searching for the roots is to be easier than directly searching for the function optimum.

Let there be a population of N trial values between the range x_{max} and x_{min} and x_{opt} is the optimum solution we are searching for. Next we compute the error using all the N values of x. Error will be of opposite sign for values of x above and below x_{opt}. Locate the trial value corresponding to minimum positive error and minimum negative error.

These values are shown against locations k and k+1 in Fig.1. Out of the two values obtained above, the lower value is assigned to xmin1 and higher value is assigned to xmax1. In the next generation all the trial values are mapped into the range between xmax1 and xmin1 Now the new search range becomes xmax1 - xmin1. The process is repeated and the trial values corresponding to minimum positive error and minimum negative error are found as xmax2 and xmin2. Hence in each generation, search range reduces and hence locating the optimum becomes much easier.

In case all the trial values are mapped either above X_{opt}or below X_{opt}, then all the error values will be of the same sign. In such a case, the mapping process is to be repeated without modifying the values of X_{min} and$X_{max..}$The reduction in search space in each generation shown in Fig.1.

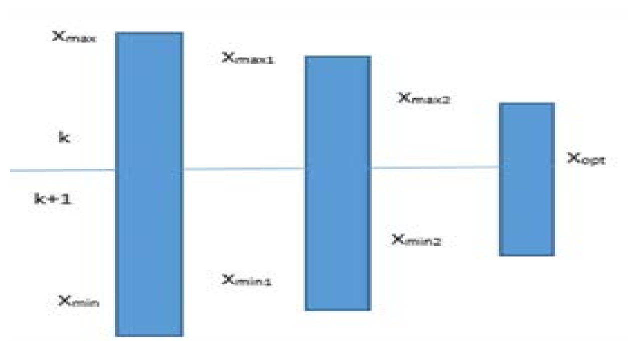

Fig 1. Search space reduction technique

II. PROBLEM FORMULATION

A.Notations

N – number of objectives
n-number of thermal units
h-number of hydro units
a_{1i} b_{1i}, c_{1i} - Cost coefficients of i^{th} unit
a_{2i}, b_{2i}, c_{2i} - NO_x emission coefficients of i^{th} unit
a_{3i}, b_{3i}, c_{3i} - SO_2 emission coefficients of i^{th}unit
a_{4i}, b_{4i}, c_{4i} – CO_2 emission coefficients of i^{th} unit
w- weight assigned to an objective (0-1)
P_{Dk}– power demand during k^{th} sub-interval
P_{Lk} – power loss during k^{th} sub-interval
P_{ik} – power output of i^{th} unit during k^{th} sub-interval
P_i^{max}, P_i^{min} – maximum/minimum power output of i^{th} unit
I- total number of sub-intervals
d_{jk} – discharge of j^{th} hydro unit during k^{th} sub-interval
λ_k – Lagrange multiplier for sub-interval k
α_j $\beta_j\gamma_j$ – discharge coefficients of j^{th} hydro unit
(Duration of a sub-interval is taken as 1 hour in this study)

B. Objective Function

The objective function is minimization of fuel cost and emission of pollutants NO_X, SO_2 and CO_2.
V_{ai} – allocated volume of water for i^{th}hydro unit
μ, φ – Lagrange multipliers

$$\text{Minimize} \quad \sum_{m=1}^{N} W_m F_m \tag{1}$$

$$\text{Where } F_m = \sum_{k=1}^{I}\sum_{i=1}^{n}(a_{mi}P_{ik}{}^2 + b_{mi}P_{ik} + c_{mi})$$

C. Constraints

1. Power balance constraint

The total generation must cover the total demand and real power loss in the transmission network

$$P_{Dk} + P_{Lk} - \sum_{i=1}^{n+h} P_{ik} = 0$$

2. Generation limit constraint

For stable operation and economic reasons each generator is restricted by upper and lower limits

$$P_i^{\min} \le P_i \le P_i^{\max} \qquad i = 1,2,3,....n+h$$

$$P_{Lk} = \sum_{i=1}^{n+h}\sum_{i=1}^{n+h} P_{ik}B_{ij}P_{jk} \quad \text{MW}$$

Using the above equations optimal power allocations for each sub-interval is found, and by applying Kuhn-Tucker conditions generation limit constraints are incorporated.
Initial set of hydro powers can be found by allocating the available quantity of water for the optimization interval over sub-intervals, proportional to power demand during each sub-interval. The remainder power demand is optimally allocated among available thermal units.

3. Weight constraint

The sum of the weights assigned to different objectives must total to 1 and it cannot have a negative value.

$$\sum_{m=1}^{N} w_m = 1$$

4. Hydro constraint

The quantity of water allocated to a hydro unit for the optimization interval must be consumed by the unit

$$\sum_{k=1}^{I} d_{jk} = Va_j, \; j = 1,2,......h \tag{2}$$

Where d_{jk} is the water discharge of the j^{th}hydro unit in k^{th} sub-interval and expressed as a quadratic in terms of discharge coefficients

$$d_{jk} = \alpha_j P_{jk}{}^2 + \beta_j P_{jk} + \chi_j \quad \text{m}^3/h$$

III. PROPOSED ALGORITHM

A. Equations Used

The approach used in this algorithm for the solution of the above problem is to find the conditions of optimality by application of calculus and solve the resulting equations by applying PSO techniques. By method of Lagrange multipliers, for a particular weight combination

$$L = \sum_{m=1}^{N} w_m F_m + \sum_{k=1}^{I}\lambda_k\left(P_{Dk} + P_{Lk} - \sum_{i=1}^{n+h}P_{ik}\right) + \sum_{k=1}^{I}\sum_{j=1}^{h}\mu_j d_{jk} - \sum_{j=1}^{h}\mu_j Va_j \tag{3}$$

Differentiation with respect to thermal and hydro powers and further simplification result in the following equations

$$P_{ik} = \left(\lambda k - \sum_{m=1}^{N} w_m b_{mi} - \lambda_k \sum_{j=1,\, j\ne i}^{n+h} 2B_{ij}P_{jk}\right) / \left(\sum_{m=1}^{N} 2w_m a_{mi} + 2\lambda_k B_{ii}\right) \quad i = 1,2,....n \tag{4}$$

$$P_{ik} = \left(\lambda_k - \mu_i\beta_i - \lambda_k \sum_{j=1,\, j\ne i}^{n+h} 2B_{ij}P_{jk}\right) / \left(2\mu_i\alpha_i + 2\lambda_k B_{ii}\right) \quad i = n+1, n+2, ..., n+h \tag{5}$$

Which are equations for power output of thermal and hydro units respectively during each sub-interval.
Losses can be computed for each sub-interval as

$$P_{Lk} = \sum_{i=1}^{n+h}\sum_{i=1}^{n+h} P_{ik}B_{ij}P_{jk} \quad \text{MW} \tag{6}$$

Using the above equations optimal power allocations for each sub-interval is found, and by applying Kuhn-Tucker conditions generation limit constraints are incorporated.
Initial set of hydro powers can be found by allocating the available quantity of water for the optimization interval over sub-intervals, proportional to power demand during each sub-interval. The remainder power demand is optimally allocated among available thermal units.

B. Find Optimum Λ By Applying Search Space Reduction Technique

The search space is defined by fixing maximum and minimum values for λ. As variations in λ is within a small range from the initially determined value by assuming losses equal to zero, the problem here is searching for the local optimum. Maximum and minimum values of λ can be fixed as; λ_{max}=1.5λ and λ_{min}=0.5λ. By fixing λ_{max} and λ_{min} as 1.5 times and 0.5 times λ respectively, a wide margin becomes available above and below λ_{opt}, which makes the search process easier.

Hence in the first trial, λ_{opt} is searched in between the above maximum and minimum values. The 'N' random generated trial values are first converted to their equivalent decimal values and then mapped between λ_{max} and λ_{min}. For each value of λ, Pik (i=1, 2,…..n+h) and $P_{Lk}$are computed using (4), (5) and (6). Error in power balance constraint for each individual in the population is found by

$$er = P_{Dk} + P_{Lk} - \sum_{i=1}^{n+h} P_{ik} \quad (7)$$

then normalized fitness is obtained by using

$$fit = 1/\left(1 + |er|/P_{Dk}\right) \quad (8)$$

Error will be of opposite sign for values of λ above and below λ_{opt} . From the N error values obtained as above, choose the positive and negative values closest to zero. Locate the corresponding values of λ and the higher value is assigned to $\lambda_{max}$and lower value to λ_{min} . Basic PSO operations are performed to get the new generation. The decimal values of strings in the new generation are mapped between new values of $\lambda_{max}$and $\lambda_{min.}$

C. Solution Approach

As our problem has four objectives, a matrix as shown in Fig.2 , is to be formed first.

1	1	0	0	0
2	0	1	0	0
3	0	0	1	0
4	0	0	0	1
5	0.5	0.5	0	0
6	0.25	0.25	0.25	0.25
7	0.50	0.25	0.05	0.20
	…………………………………… ……………………………………			
L				

Fig. 2 A set of L weight vectors for a four objective problem

The matrix has L rows and N columns (in this case 4 columns) where L has a standard value, normally 20 or 30 and N is the number of objectives in the problem. All the N elements of a row should be in the range 0-1 and also they should add up to 1. But the first N elements of the matrix are predefined such that one of the objectives get a weight of 1.0 and the remaining objectives get a weight of zero each such that the extreme values all the objective functions Fi_{max} and Fi_{min} can be determined.

Another matrix of L rows and 1 column is formed for finding the optimum value of λ in each sub-interval. Optimum λ is found using search space reduction technique. Corresponding values of power and losses are evaluated for each sub-interval.

Withdrawal of water (V_j, j=1,2,….,h) is computed at the end of optimization interval and if conditions of (1) are not satisfied then the values of μ_j (j=1,2,…..,h) are modified as shown in (9) and the iterations are continued.

$$\mu_j \leftarrow \mu_j + \nabla\mu_j \quad (9)$$

where $\nabla\mu_j = k\mu_j (V_j - Va_j)/Va_j$

The number of iterations for satisfying hydro constraints has a strong dependence on acceleration factor k. The value of k should be carefully chosen and modified. An optimum value of k in each iteration can be determined using fuzzy techniques which reduces iterations and time by 30%.

If hydro constraints are satisfied, the solution converges for a particular weight combination and the total cost and total emission of each pollutant for the optimization interval are found. The membership function of each objective is evaluated as

$$m(F_i) = (F_i^{\max} - F_i) / F_i^{\max} - F_i^{\min}) \quad (10)$$

$$where\ F_i^{min} \le Fi \le F_i^{min}$$

$$m(F_i) = 1\ for\ F_i \le F_i^{min}\quad for\ \ i = 1,2,...,N$$

$$m(F_i) = 0\ for\ F_i \ge F_i^{\max}\quad for\ i = 1,2,....,N$$

$$fitness = \sum_{i=1}^{N} m(F_i) / N \quad (11)$$

'L' trials are made with the first population after which it is modified. The algorithm continues till there is hardly any improvement in the highest
fitness value obtained.

D. PSO Operations Incorporated

In this problem PSO is used at two levels, for determining optimum in each optimization sub-interval and for generating and modifying weight combinations.
Search for λ is a search for local optimum whereas search for a weight combination is a search for global optimum. In both the case the trial values and are stored in matrices of sizes Lx1 and Lx4 and the best values are stored in matrices of the same size.

While searching for λ, search space reduction technique is applied in each iteration to modify the upper and lower limits of the search range. Thus in each generation, the search range reduces. The present values of λ are mapped to the new range and each value is modified using (12).

$$\lambda_{\text{mod}} = w \times \lambda + C_1 \times rand1 \times (\lambda_{pbest} - \lambda) + C2 \times rand2 \times (\lambda_{gbest} - \lambda) \quad (12)$$

The modified value of a parameter is the sum of three terms which are the following.

1. Its present value multiplied by a weighting function
2. The product of three parameters: a constant, the difference between its personal best value and the present value and a random number less than one

3. The product of three parameters: a constant, the difference between its global best value and the present value and a random number less than one

When a parameter is modified, it is desired that its value should not change drastically as it is a local search. The value of w, C_1 and C_2 may be selected accordingly. Here fuzzy logic techniques are used to determine C_1, C_2 and w.

VI. STRENGTH OF THE PROPOSED ALGORITHM

Partial application of calculus simplifies the method a lot. A direct search for a set of powers, satisfying the constraints and minimizing the composite objective function value, is rather time consuming. Search for a single parameter λ, has become extremely fast by the application of search space reduction technique, though the algorithm uses only basic operations.

Normally the weight combinations are user defined. Also it can be auto-generated. If the weight combination is user defined, the major difficulty encountered is to accurately define the weights such that a high fitness value is obtained. Highly promising results are obtained when weight combinations are PSO modified.

V. TEST DATA

The algorithm is tested on a four generator system consisting of two thermal and two hydro units. Cost, emission discharge and loss coefficients are given in Table I to III. Table IV gives water allocation and Table V gives generation limits of thermal generators.

TABLE I FUEL COST AND EMISSION COEFFICIENTS

	Thermal unit 1	Thermal unit 2	
a1	0.0025	0.0008	Fuel cost
b1	3.20	3.400	
c1	25.0	30.0	
a2	0.006483	0.006483	NO_x emission
b2	-0.79027	-0.79027	
c2	28.82488	28.82488	
a3	0.00232	0.00232	SO_2 emission
b3	3.84632	3.84632	
c3	182.26	182.26	
a4	0.084025	0.084025	CO_2 emission
b4	-2.94458	-2.94458	
c4	137.7043	137.7043	

TABLE II HYDRO DISCHARGE COEFFICIENTS

	Hydro unit 1	Hydro unit 2
α	0.06	0.065
β	20.0	22.5
γ	140.0	150.0

TABLE III LOSS COEFFICIENTS B_{ij}

	1	2	3	4
1	0.000140	0.000010	0.000015	0.000015
2	0.000010	0.000060	0.000010	0.000013
3	0.000015	0.000010	0.000068	0.000065
4	0.000015	0.000013	0.000065	0.000070

TABLE IV TOTAL WATER ALLOCATION FOR ALL HYDRO PLANTS (M^3)

Hydro unit 1	Hydro unit 2
100000.0	110000.0

TABLE V GENERATION LIMITS

	Thermal unit 1	Thermal unit 2	Thermal unit 3	Thermal unit 4
Min. (MW)	60.0	80.0	50.0	55.0
Max. (MW)	800.0	1000.0	600.0	500.0

Power demand varies from 525 MW to 1470 MW in various sub-intervals.

VI. RESULTS

The first four weight combinations and their fitness values are given in Table 6 and the corresponding values of objective functions are given in Table 7. For the first objective function F_1^{max}= 76265.2 and F_1^{min}=75585.0. Similarly, F_2^{max}=45790.2 and F_2^{min}=43370.5. The fitness value obtained is the lowest for the first weight combination and highest for the third weight combination.

Final results of a trial are shown in Table VIII. Weight combinations and corresponding fitness values as shown are obtained after 10 generations. Table shows only the first eight elements, sorted in the descending order of fitness.

TABLE VIFITNESS VALUES OF FIRST FOUR WEIGHT COMBINATIONS

S. No.	W_1	W_2	W_3	W_4	fitness
1	1	0	0	0	0.341967
2	0	1	0	0	0.521539
3	0	0	1	0	0.739271
4	0	0	0	1	0.497713

TABLE VII VALUES OF OBJECTIVE FUNCTIONS FOR WEIGHT COMBINATIONS GIVEN IN TABLE VI

S. No.	F_1(Rs.)	F_2(kg)	F_3(kg)	F_4(kg)
1	75585.0	45790.2	103072.4	720774.1
2	76245.4	43370.5	103234.9	691421.4
3	75767.0	44340.4	102742.3	702307.6
4	76265.2	43379.0	103263.9	691383.5

TABLE VII WEIGHT COMBINATIONS AND CORRESPONDING FITNESS VALUES OBTAINED AFTER 10 GENERATIONS IN A TRIAL RUN

	W_1	W_2	W_3	W_4	fitness
1	0.2438	0.1701	0.5862	0	0.7868
2	0.2551	0.1149	0.6250	0.0051	0.7866
3	0.2179	0.0866	0.6898	0.0060	0.7848
4	0.2475	0.1072	0.6327	0.0128	0.7845
5	0.3100	0.0575	0.6250	0.0075	0.7834
6	0.2646	0.1121	0.6092	0.0146	0.7824
7	0.2879	0.0968	0.6129	0.0025	0.7810
8	0.2454	0.1201	0.6348	0	0.7806

VII. CONCLUSION

In this paper a novel approach is devised to solve the short term hydrothermal scheduling problem. Feeding a set of weight combinations need not identify high fitness solutions. On such an occasion the algorithm described in this paper is highly promising. As the algorithm retains all good solutions, a choice can be made among them. From the solution matrix one can pick up a solution depending on the need. For example, if cost is to be given full priority then the solution corresponding to the first weight vector in Table VI can be chosen. If best compromise solution is the one required, then the first solution vector in Table VIII is to be considered. Table VIII gives a set of eight high fitness solutions from which a simple selection can be made depending on one's priority.

As new techniques are incorporated, the solution is faster and also very simple.

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