Thermodynamics of the First Order Phase Transition of Quark-Gluon Plasma at RHIC

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Abstract - **The quark-gluon plasma in thermodynamic equilibrium is reviewed. The phase diagram of quark hadron phase transition and the color superconducting phases of quark-gluon matter are discussed. The lattice QCD results on the order of the phase transition. We discuss some aspects of atomic Fermi gas in the unitary limit in the first order transition. We consider the equation of state and the critical temperature for pair condensation. The strongly interacting phase transition (Quark-Hadron phase transition) also discussed.**
Key words: **Quark chemical potential, Baryon density, Lattice QCD, Equation of state.**

I.INTRODUCTION

The quark gluon plasma (QGP) is a novel state of matter conjectured to form during ultra relativistic heavy-ion collisions at CERN and RHIC. LHC at CERN seek to create this state in collisions of heavy ions such as Pb and Au up to $\sqrt{S} \sim$ 200 GeV/nucleon at RHIC and 5 TeV/nucleon at LHC. A primary goal of relativistic nuclear collisions is the observation of a phase transition of confined, hadronic matter to deconfined quark-gluon plasma. One of the proposed signatures, namely hydrodynamic flow, is based on the presumed softening of the eqution of state due to the rapid increase of the entropy density. Current theoretical ideas suggest that a QGP in local thermodynamic equilibrium is formed on a time scale of the order of $\sim 1fm = c$. In the QGP, mesons and baryons lose their identities and make a much larger mass of quarks and gluons. In normal matter quarks are confined; in the QGP quarks are deconfined. Lattice QCD calculations have established the existence of such a phase of strongly interacting matter at temperature larger than $\sim$ 150 MeV and zero net-baryon density. Depending upon masses of the quarks and number of quark flavors, the transition between ordinary hadron matter and the QGP could be a thermodynamic phase transition of first order, of second order or a crossover transition.

The order of a phase transition is a basic thermodynamic characteristic. A phase transition is said to be of first order if there is at least one finite gap in the first derivatives of a suitable thermodynamic potential in the thermodynamic limit. A transition is said to be of second order if there is a power-like singularity in at least one of the second derivatives of the potential. The ground state of (infinite) nuclear matter is at $(T,\mu)_0 = (0,308)$ MeV. There is a line of first order phase transitions emerging from this point and terminating at a critical endpoint at a temperature of the order of $\sim$ 10 MeV. At this point, the transition is of second order. This phase transition is the nuclear liquid-gas transition [2]. The liquid-gas transition in nuclear matter is a consequence of the fact that nuclear matter assumes its ground state at a nonvanishing baryon density $n_{B,0} \sim 0.17$ fm^{-3} at zero temperature T = 0. For temperature below $\sim$ 170 MeV and quark chemical potentials $\mu <$ 350 MeV, strongly interacting matter is in the hadronic phase. Similar to the liquid-gas transition, there is a line of first-order phase transition which separates the hadronic phase from the QGP and terminates in a critical end point at $(T,\mu) \sim$ (170, 240) where the transition is of second order. For smaller quark chemical potentials or smaller net-baryon densities, the transition becomes crossover, and there is no real distinction between hadronic matter and the QGP. As is well-known, fluctuations are large for statistical systems near their critical points. Thus the study of fluctuations in the process might reveal some features for the phase transition [1, 3]. Finally at large quark chemical potential (large baryon density) and small temperature, quark matter becomes a color condensate. For quark chemical potentials μ which are on the order of 350 MeV or smaller, and for temperatures $T < \Lambda_{QCD} \sim$ 200 MeV, nuclear matter is a gas of hadrons. (At very small temperatures $T <$ 10 MeV, there is a gaseous and a liquid nucleonic phase. On the other hand, for T, $\mu >> \Lambda_{QCD}$, nuclear matter consists of quarks and gluons.

The latest RHIC experimental data is shown that the QGP is indicated as strongly coupled quark-gluon liquid of very low viscosity rather than a gas of weakly interacting quarks and gluons [4]. Therefore QGP-hadrons phase transition should be a first order phase transition of the liquid-gas type. In present paper, the quark-gluon plasma is described phenomenologically and the transition from QGP to hadrons is considered as a first-order phase transition. Ginzburg-Landau (GL) formalism and the hysteresis phenomenon is typical for the first-order phase transitions [5].

II.LIQUID-GAS PHASE TRANSITION

Nuclear matter shows a behavior, featuring a "gaseous" phase of nucleons at small chemical potentials (densities) and a "liquid" phase of nuclear matter at large chemical potentials (densities). At small temperatures the transition between the two phases is of first order. In infinite, isospin-symmetric nuclear matter, nucleons in the ground state are bound by -16 MeV (by neglecting Coulomb repulsion) i.e., the energy per baryon is $(\epsilon/n_B)_0 \cong 924 \, MeV$ where ϵ is the energy density and n_B is the baryon density. Nuclear matter is mechanically stable in the ground state, such that the pressure vanishes i.e. $p = 0$. From the fundamental relation of thermodynamics, $\epsilon = Ts + \mu n - p$, it is clear that the baryon chemical potential in the ground state is identical to the energy per baryon.

For making baryon density $n_B > n_{B,0}$, energy consumed, leading to the increase in pressure. Since pressure vanishes in the ground state and is a monotonous function of the thermodynamic variables, there are only two possibilities for the behavior of p for densities $n_B < n_{B,0}$; either the pressure remains zero as the density decreases, $p = 0$, or the

pressure further decreases such that $p < 0$. But the last possibility makes the system unstable and therefore fragmenting the nuclear matter into droplets because each droplet is mechanically stable. The density inside each droplet is equal to $n_{B,0}$ and the pressure vanishes. The total baryon density in the system can be further reduced by simply decreasing the droplet density. The pressure in such a system remains zero down to arbitrarily small densities. At small temperatures and densities below the ground state density, one thus has a mixed phase of nucleons and droplets of nuclear matter. By changing the density, one can change the relative fraction of molecules and droplets. Beyond the density where droplets fill the entire volume one enters the liquid phase, while below the density where the last droplet fragments into molecules one enters the gas phase. This behavior is typical for a first-order phase transition of liquid-gas transition type in water.

Nuclear matter shows a similar behavior, featuring a "gaseous" phase of nucleons at small chemical potentials (densities) and a "liquid" phase of nuclear matter at large chemical potentials (densities), Fig 1. At small temperatures the transition between the two phases is of first order.
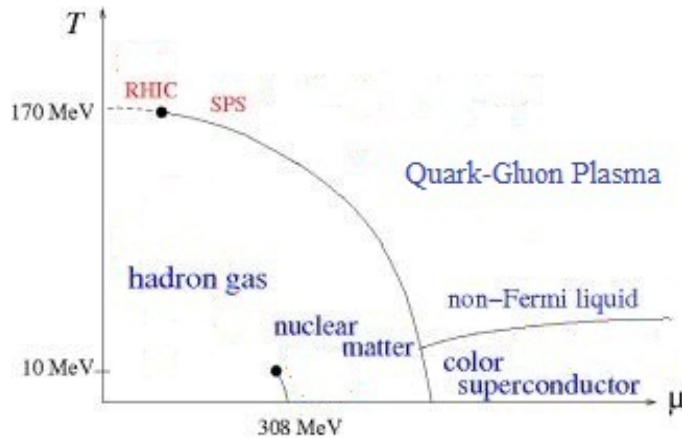

Fig.1The phase diagram of strongly interacting matter (schematic).

In (T, μ) phase diagram, there is a line of first-order phase transitions extending from the nuclear ground state (0,308) MeV up towards larger values of T and smaller values of μ. The critical temperature is of the order of 10 MeV. One cannot distinguish between the gaseous and the liquid phase for temperatures above this critical temperature.
Ideal fluid hydrodynamics has been particularly successful in describing the observed anisotropy of particles at low p_T in minimum bias collisions [6, 7]. Studies of both single particle spectra and anisotropies have also shown that a reasonable reproduction of data favors an Equation of State (EoS) of strongly interacting matter with a phase transition [8, 9]. The lattice QCD calculations of the EoS of strongly interacting matter support such a scenario by predicting a phase transition at $T_c \approx 170 \, MeV$ temperature. For a physical scenario of two lights and one heavier quark, the

phase transition is predicted to be a smooth crossover at small values of baryochemical potential. At mid-rapidity at collisions at RHIC, the net baryon density is small and the relevant EoS should exhibit a crossover transition.

III.THE QUARK-HADRON PHASE TRANSITION

Particles can only scatter elastically if their momenta lie on the Fermi surface. If the Fermi momentum exceeds the QCD scale parameter $\Lambda_{QCD} \sim 200 \, MeV$, scattering events between nucleons start to probe distances of the order 1 fm or less. In this way, the nucleonic substructure of quarks and gluons becomes visible. The Fermi momentum in the ground state of nuclear matter can be ~ 200 MeV. At nonzero temperature, nuclear matter consists of nucleons as

well as thermally excited hadrons. For a non-interacting system in thermodynamically equilibrium, the hadron density is proportional to $n_i \sim m_i^2 T K_2(m_i/T) e^{\mu_i/T}$, where i labels the hadron species, m_i and μ_i are their mass and chemical potentials respectively, K_2 is the modified Bessel function of second order. For quark chemical potentials μ which are on the order of 350 MeV or smaller and for temperatures $T < \Lambda_{QCD} \sim 200\,MeV$, nuclear matter is a gas of hadrons. If the temperature is of the order of (or larger than)Λ_{QCD}, the scattering between hadrons starts to probe their quark-gluon substructure. For $\mu \gg \Lambda_{QCD}$, nuclear matter consists of quarks and gluons. Moreover, since the particle density increases with the temperature, the hadronic wave functions will start to overlap for large temperatures. Consequently, above a certain temperature one expects a description of nuclear matter in terms of quark and gluon degrees of freedom to be more appropriate.

IV.TRANSPORT PROPERTIES AND EQUATION OF STATE

Experiments at RHIC indicate that the viscosity of plasma is very small. The crossover from weak to strong coupling can also be studied corresponding to the cold atomic gas. In the non-interacting limit, the energy per particle is $E/N = 3E_F/5$. The Fermi energy $E_F = k_F^2/2m$ is related to the energy density $N/V = k_F^3/3\pi^2$. Asymptotic freedom implies that the equation of state of quark-gluon plasma at $T \gg \Lambda_{QCD}$ is that of a free gas quarks and gluons. Lattice QCD calculations show that at $T \sim 2T_c$ (which is about the early stage of heavy ion collisions at RHIC), the pressure and energy density reach about 80% of the free gas limit. This is consistent with the first order perturbation correction. The matter so formed at RHIC is characterized by strong radial and elliptical flow [10]. This result shows that the shear viscosity to entropy ratio of QGP at temperature near T_c must be very small [11] i.e. $\eta/s \ll 1$. In a weak coupling limit, the shear viscosity to entropy density ratio in perturbative QCD is [12]

$$\frac{\eta}{s} = \frac{5.12}{g^4 \log(2.42g^{-1})}$$

In a weal coupled ($g \sim 1$) QGP, the ratio η/s is very large. This ratio decreases as the coupling increases. In the cold atomic gas, η/s can be compute in the BCS limit. The ratio is temperature dependent and has a minimum at $T \sim T_F$, where T_F is the Fermi temperature and is equal to $T_F = E_F/k_B$.

In high baryon density and low temperature, the QCD phase diagram contains a number of color superconducting phases, which is characterized by the formation of Cooper Pairs. In this limit, the pairing gap is given by

$$\Delta = 2\Lambda_{BCS} \exp\left(-\frac{\pi^2 + 4}{8}\right) \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right).$$

Where g is the coupling constant, and $\Lambda_{BCS} = 256\pi^4 (2/N_f)^{5/2} g^{-5}\mu$. Here N_f is the number of flavors. In weal coupling limit, the gap and the critical temperature are exponentially small. The ratio T_c/E_F increases with g and reaches a maximum at $g = 4.2$. At low temperature, the atomic gas becomes super fluid and is occurs when $k_B T_c \cong 0.15 E_F$. If the coupling is weak, the gap and the critical temperature can be calculated by using BCS theory. In BCS theory, the critical temperature is given by $T_c = e^\gamma \Delta/\pi$. Thus the critical temperature depends upon the scattering length.

V.CONCLUSION

In this review, I have presented the current knowledge of the first order phase transition of interacting QGP matter at varying temperatures and/or densities. In particular, I have qualitatively discussed the phase diagram. I have presented calculations of thermodynamic properties of strongly interacting matter, both via lattice QCD, as well as within analytic approaches. Finally, I have given an overview of color superconductivity in weak coupling corresponding to BCS limit. At small temperatures and densities below the ground state density, one thus has a mixed phase of nucleons. At large quark chemical potential (large baryon density) and small temperature, quark matter becomes a color condensate. At low temperature, the atomic gas becomes super fluid and is occurs when $k_B T_c \cong 0.15 E_F$.

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