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A Study on the Stochastic Approximation Method during the Single Server Queueing System with Catastrophe

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***Abstract* - We consider an M/G/1 queueing system where the customer arrivals occur according to a Poisson process with mean arrival rate λ. The server is subject to catastrophe and repairs while in operation. At failure times, the server still works at a lower service rate rather than completely stopping service.The service time follow general laws with probability distribution function and Laplace Stieltjes transform. Using the approximation conditions in the classical M/G/1 system, we obtain stability inequalities with exact computation of the constants.**

***Keywords:* Queueing system, Steady state, catastrophe, approximation condition, stability condition, Mean queue length.**

I.INTRODUCTION

The simplest queueing models with catastrophes have been studied some years ago, see the motivation and first results in [1,5]. Here we consider the essentially general situation of using the stochastic approximation conditionon the single server queuing system with catastrophe and delayed repair to obtain.

Stochastic population models with catastrophes, or catastrophes that model partial or nearly total extinctions have been studied for a long time [3]. Among the first papers that considered the effect of catastrophes to queueing processes were Gelenbe, Glynn, and Sigman [9], which considers queues with negative customers (of which catastrophes are a special case), Chao [4] which considers a Jackson network in the presence of catastrophes and shows that its stationary distribution retains the product form, and Jain and Sigman [12] which considers the M/G/1 system under Poisson catastrophes and generalizes the Pollaczek–Khintchine formula for the steady state workload. The stationary distribution of the workload is also analyzed in Boxma et al. [2] for M/G/1 queues via a martingale argument for various clearing (i.e. catastrophe) rules.

Since then queueing systems with catastrophes have been recognized as useful and natural models for communications systems and manufacturing operations subject to catastrophic failures. Dubin and Nishimura [6] examine a single server queue in the more general framework of customers arriving according to a Batch Markovian Arrival Process with catastrophes occurring according to a Markovian Arrival Process and obtain the queue length and the sojourn time distribution. In [7] this model is extended to include repair times after the occurrence of catastrophes. The Laplace transform for the busy period and sojourn times of an M/G/1 queue system with catastrophes followed by repair times was studied in Yang et al. [16]. Yechiali [15] studies an M/M/c queue subject to catastrophes and subsequent repair periods, during which arriving customers wait in line but abandon the system at an exponential rate. A variation of this model for the M/M/1 queue where customers perform synchronized abandonments was studied by Economou and Kapodistria [8]. Retrial queues with catastrophes were investigated by Artalejo and Corral [1] using the supplementary variables technique. Shin [14] considers multi-server retrial queues with an MMAP arrival process in the presence of catastrophes and negative customers and provides an algorithmic solution for the stationary queue length distribution using matrix geometric techniques. The transient behavior of the M/M/1 queue with catastrophes, followed by exponential repair periods is studied in Kumar et al. [11]. Therefore, analytic closed-form solutions could be obtained only for certain special unreliable models. Nevertheless, even in these cases, the complexity of these analytical formulas does not allow exploiting them in the practice, that is the case of the generating function of the various limiting distributions or Laplace transforms [14]. For this reason, there exists, when modelling a real system, a common technique for substituting the real but complicated elements governing a queueing system by simpler ones in some sense close to the real elements. A queueing system is said to be stable if small perturbations in its parameters entail small perturbations in its characteristics [15, 16]. So, the margin between the corresponding characteristics of two stable queueing systems is obtained as function of the margin between their parameters. In contrast to other methods, we assume that the perturbations of the transition kernel are small with respect to some norms in the operator space. Such a strict condition allows us to obtain better stability estimates and enables us to find precise asymptotic expressions of the characteristics of the perturbed system [16]. The basic model of a population size subject to occurrence Catastrophical events and the quantal response in connection with the population is studied in Reni sagayaraj and Anand et al.[12]. The Analysis of

Transient behavior M/M/1 Queueing model with Catastrophical events is studied in Reni Sagayaraj and Anand et al. [13].

This paper is organized as follows. The Mathematical description of ourmodel is given in section 2. In section 3stability condition has been used to obtain steady state results inexplicit and closed form in terms of the probability generating functions forthe number of customers in the queue.The mean number of customer in thequeue as well as in the system has been found in section 4.In section 5 Conclusion.

II. MODELS DESCRIPTION AND ASSUMPTIONS

We consider an M/G/1queueing system (FIFO;∞) with a repairable service station. Customers arrive according to a Poisson process with rate λ¸ and demand an iid service times with common distribution function B with mean 1/μ. We shall assume that when a server fails, the time required to repair it have exponential distribution with rate r > 0.In this model, we consider the breakdowns with losses. If an arriving customer finds the server idle-up, it will be immediately occupying the server with probability q > 0and leaves it after service completion if no breakdown had occurred during this period. If the server is failed, then this blocked customer may leave the system altogether without being served with probability 1- q. We assume that when the server fails, it is under repair and it cannot be occupied. As soon as the repair of the failed server is completed, the server enters an operating state and continues to serve the other customers immediately.

The state of M/G/1 queueing system with a repairable service station at time t, can be described by the following stochastic process,

$$S(t)=\{X(t), j(t), C(t); t\geq 0\} \qquad (2.1)$$

where $X(t)\in N$ is the number of customers in the system at time t and $\{e(t), K(t)\}$ are two supplementary state variables, $e(t)\in\{0,1\}$ describes global properties of the server:

$$e(t)=\begin{cases} 0, \text{ if the server is available for service at time t} \\ 1, \text{ if the server is failed at time t} \end{cases} \qquad (2.2)$$

K(t) is a random variable, which is defined below.

- If e(t) = 0 and X(t) = 0, then K(t) is equal to the time elapsed since the time t to the occurrence time of a Catastrophe, if after t the arrivals are interrupted.
- If X(t) ≠ 0.
- If e(t) = 0, K(t) is the duration of the remaining service period.
- If e(t) = 1, K(t) is the duration of the remaining repair period.

Clearly, S(t) is a Markov process. The computation of its transitional regime intervenes nth integro-differential equations. For that, we use the method of the embedded Markov chain.
We use the following notations.

γ_n are the times of end of the nth service.

Δ_n is a random variable, that represents the number of the arrivals during the nth service time.

$X_n = X(\gamma_n)$ is number of customers in the system immediately after the end of the nth service time.

$$\delta_{X_n}=\begin{cases} 0, \text{If } X_n = 0 \\ 1, \text{ If } X_n > 0 \end{cases} \qquad (2.3)$$

$$\Pi_{\{e_n=1\}}=\begin{cases} 0, \text{If } e_n = k \\ 1, \text{ Otherwise} \end{cases} \qquad (2.4)$$

Where $k\in\{0,1\}$

Lemma: 2.1

The sequence $S_n = S(\gamma_n)$ Forms a markov chain, its transition operator $P=\left(p_{ij}\right)_{i,j\geq 0}$ is defined by

$$p_{ij} = \begin{cases} q\int_0^\infty e^{-\lambda x\frac{(\lambda x)^j}{j!}dB(x)+\frac{r(1-q)}{\lambda+r}\left(\frac{\lambda}{\lambda+r}\right)^j} & \text{if } i=0 \\ q\int_0^\infty e^{-\lambda x\frac{(\lambda x)^{j-i+1}}{(j-i+1)!}dB(x)+\frac{r(1-q)}{\lambda+r}\left(\frac{\lambda}{\lambda+r}\right)^{j-i+1}} & \text{if } 0<i\le j+1 \end{cases} \tag{2.5}$$

Proof: To calculate the probabilities $\left(p_{ij}\right)_{i,j\ge 0}$ we consider all the following possible events.

$$1)\left\{X_{n+1}=j \text{ and } e_{n+1}=0\right\} \text{given that} \left\{X_n=i \text{ and } e_n=0\right\}; i,j\ge 0.$$
$$2)\left\{X_{n+1}=j \text{ and } e_{n+1}=1\right\} \text{given that} \left\{X_n=i \text{ and } e_n=0\right\}; i,j\ge 0. \tag{2.6}$$
$$3)\left\{X_{n+1}=j \text{ and } e_{n+1}=0\right\} \text{given that} \left\{X_n=i \text{ and } e_n=1\right\}; i,j\ge 0.$$

We have

$$X_{n+1}=X_n+\Delta_{n+1}\Pi_{\{e_{n+1=k}\}}-\delta_{X_n}; k=0,1.$$

Therefore the transient probabilities p_{ij} are written under the following form.

$$\begin{aligned} p_{ij} &= \Pr[X_n+\Delta_{n+1}\Pi_{\{e_{n+1=0}\}}-\delta_{X_n}=j/X_n=i]+\Pr[X_n+\Delta_{n+1}\Pi_{\{e_{n+1=1}\}}-\delta_{X_n}=j/X_n=i] \\ &= q\Pr[\Delta_{n+1}\Pi_{\{e_{n+1=0}\}}=j-i+\delta_{X_n}]+(1-q)\Pr[\Delta_{n+1}\Pi_{\{e_{n+1=1}\}}=j-i+\delta_{X_n}] \end{aligned}$$

If

$X_n = i \neq 0,$ then

$$P_{ij}=q\Pr[\Delta_{n+1}\Pi_{\{e_{n+1=0}\}}=j-i+1]+(1-q)\Pr[\Delta_{n+1}\Pi_{\{e_{n+1=0}\}}=j-i+1] \tag{2.7}$$

If $X_n = i = 0,$ then

$$P_{0j}=\text{qPr}[\Delta_{n+1}\Pi_{\left(e_{n+1=0}\right)}=\text{j}]+\text{(1-q)Pr}[\Delta_{n+1}\Pi_{\left(e_{n+1=1}\right)}=\text{j}] \tag{2.8}$$

Where the distribution of Δ_n Is given by

If $e_n = 0$ then

$$\text{pr}[\Delta_n\Pi_{(e_n=0)}=k]=\int_0^\infty e^{-\lambda x}\frac{(\lambda x)^k}{k!}\Pi_{\{e_n=0\}}dB(x) \tag{2.9}$$

If $e_n = 1$ then

$$\begin{aligned} \text{pr}[\Delta_n\Pi_{(e_n=1)}=k] &= r\int_0^\infty e^{-(\lambda+r)x}\frac{(\lambda x)^k}{k!}\Pi_{\{e_n=1\}}dB(x) \\ &= \frac{r}{\lambda+r}\left(\frac{\lambda}{\lambda+r}\right)^k \end{aligned}$$

To calculate the operator's transition, we distinguish using the following cases.

Case:1

If the system is empty that is $X_n = i = 0$ then,

$$p_{oj}=q\int_0^\infty e^{-\lambda x}\frac{(\lambda x)^j}{j!}\Pi_{\{e_{n+1=0}\}}dB(x)+\frac{r(1-q)}{\lambda+r}\left(\frac{\lambda}{\lambda+r}\right)^j \tag{2.10}$$

Case:2

If there are no arrivals during the service duration of $(n+1)^{th}$ customers duration of the server just after the departure of the nth customer, then

$$p_{ij} = q\int_0^\infty e^{-\lambda x}\frac{(\lambda x)^j}{j!}\Pi_{\{e_{n+1=0}\}}dB(x) + \frac{r(1-q)}{\lambda + r}$$

Casse:3

If $j > i-1$ then

$$p_{ij} = q\int_0^\infty e^{-\lambda x}\frac{(\lambda x)^{j-i+1}}{(j-i+1)!}\Pi_{\{e_{n+1=0}\}}dB(x) + \frac{r(1-q)}{\lambda + r}\left(\frac{\lambda}{\lambda + r}\right)^{j-i+1}$$

Casse:4

If $j < i-1$ then $p_{ij} = 0$

Consider also an M/G/1 (FIFO;∞) system with null catastrophes rate. It behaves exactly as a classical M/G/1 system: Arrivals occur as a Poisson process of rate and it has the same general service time distribution function B. Let $\bar{P}$ be the transition operator for the corresponding Markov chain $\bar{X}_n$ in the classical M/G/1 system. We have [16].

$$\bar{P}_{ij} = \begin{cases} \int_0^\infty \frac{(\lambda x)^j}{j!}e^{-\lambda x}dB(x), & \text{if } j \geq 0, i=0; \\ \int_0^\infty \frac{(\lambda x)^{j-i+1}}{(j-i+1)!}e^{-\lambda x}dB(x), & \text{if } 1 \leq i \geq j+1; \\ 0, & \text{otherwise} \end{cases} \quad (2.11)$$

We propose to obtain the stability inequalities and this by applying the strong stability criterion [14] for the M/G/1 queueing system with null catastrophe's rate. Indeed, the approximation conditions in the classical M/G/1 queueing system have been made [10] at the time of perturbation of the retrial's parameter in the M/G/1/1 retrial queue. By making tend the retrial' state to 1 and the resultant system is exactly the M/G/1 system with infinite retrials. Also, in our case, we make tend the catastrophe rate to zero in the M/G/1 with catastrophe and repairs, so we will have the classical M/G/1 system. Therefore, we can consider the two perturbed systems, the M/G/1with repairable station and the retrial M/G/1 queue, as two sequences which have the same limit.

III. THE STABILITY CONDITION

Let $M = \{\mu_j\}$ be the space of finite measures on N, and $\eta = \{f(j)\}$ the space of bounded measurable functions on N. We associate with each transition kernel P the linear mapping:

$$(\mu P)_k(j) = \sum_{i\geq 0}\mu_i P_{ik}(j) \quad (3.1)$$

$$Pf(k) = \sum_{i\geq 0} f(i)P_{ki} \quad (3.2)$$

Introduce on M the class of norms of the form

$$\|\mu\|_v = \sum_{i\geq 0} v(j)|\mu_j| \quad (3.3)$$

Where v is an arbitrary function bounded below away from a positive constant. The norm induces in the space η the form

$$\|f\|_v = \sup_{k\geq 0}\frac{|f(k)|}{v(k)} \quad (3.4)$$

Let us B, the space of linear operators with norm

$$\|P\|_v = \sup_{k\geq 0}\frac{1}{v(k)}\sum_{j\geq 0} v(j)|P_{kj}| \quad (3.5)$$

Stability inequalities in the classical M/G/1 queue:

To able to estimate numerically the margin between the stationary distributions of the markov chains $\bar{X}_n$ and X_n . We estimate the norm of the deviation of the transition kernel $\bar{P}$. And we use the following lemma.

Lemma 3.1: Let us suppose that $\int_0^\infty x\left|B-E\right|(dx)<\frac{W}{\lambda}$ Where $W=W(B,E)=\int_0^\infty \left|E-b\right|(dx)$ and that it exists a>0 such as $\int_0^\infty e^{ax}\left|B-E\right|(dx)<\infty$. Then it exists $\beta>1$ such as

$$\int_0^\infty e^{\lambda(\beta-1)x}\left|B-E\right|(dx)<\beta W \qquad (3.6)$$

Theorem 3.1. Let P be the transition operator of the embedded Markov chain in the M/G/1 queueing system with catastrophe and repairs . Let us suppose that the conditions of the Lemma (3.1) are verified. Then, for all β such that $1<\beta<\beta_0$. We have

$$\left\|P-\bar{P}\right\|_v \leq \beta_0 w \qquad (3.7)$$

Where $W=W(B,E)=\int_0^\infty \left|B-E\right|(dx)$ and $\beta_0=Sup\left\{\beta;\frac{\bar{f}(\lambda\beta-\lambda)}{\beta}<1\right\}$

Proof:

From the equation (16), then we have

$$\left\|P-\bar{P}\right\|_v =\sup_{k\geq 0}\frac{1}{\beta^k}\sum_{j\geq 0}\beta^j\left|P_{kj}-\bar{P}_{kj}\right| \qquad (3.8)$$

i) For K=0

$$\left\|P-\bar{P}\right\|_v = \sum_{k\geq 0}\beta^k\left|P_{0k}-\bar{P}_{0k}\right|$$

$$= (1-q)\sum_{k\geq 0}\beta^k\left|\int_0^\infty e^{-\lambda x}\frac{(\lambda x)^k}{k!}dE(x)-\int_0^\infty e^{-\lambda x}\frac{(\lambda x)^k}{k!}dB(x)\right|$$

$$\leq \sum_{k\geq 0}\beta^k\int_0^\infty e^{-\lambda x}\frac{(\lambda x)^k}{k!}\left|B-E\right|(dx)$$

$$\leq \int_0^\infty e^{-\lambda x}\sum_{k\geq 0}\beta^k\frac{(\lambda x)^k}{k!}\left|B-E\right|(dx)$$

$$\leq \int_0^\infty e^{-\lambda(\beta-1)x}\left|B-E\right|(dx)$$

$$\leq \beta W$$

ii) For $k \neq 1$

$$\left\| P - \bar{P} \right\|_v = \sup_{k \geq 0} \frac{1}{\beta^k} \sum_{j \geq 0} \beta^j \left| P_{kj} - \bar{P}_{kj} \right|$$

$$\leq \sup_{k \geq 0} \frac{1}{\beta} \sum_{j \geq k-1} \beta^{j-k+1} \int_0^\infty e^{-\lambda x} \frac{(\lambda x)^{j-k+1}}{(j-k+1)!} \left| B - E \right| (dx)$$

$$\leq \frac{1}{\beta} \int_0^\infty e^{-\lambda x} \sum_{j \geq 0} \frac{(\lambda \beta x)^j}{j!} \left| B - E \right| (dx)$$

$$= \frac{1}{\beta} \int_0^\infty e^{-\lambda(\beta-1)x} \left| B - E \right| (dx)$$

Then we have

$$\left\| P - \bar{P} \right\|_v \leq \frac{1}{\beta^k} \int_0^\infty e^{\lambda(\beta-1)x} \left| B - E \right| (dx) \qquad (3.9)$$

Now $\int_0^\infty e^{\lambda(\beta-1)x} \left| B - E \right| (dx) < \beta W$ (3.10)

From which $\left\| P - \bar{P} \right\|_v \leq W$ (3.11)

According to the above Lemma (3.1), We will have

$$\left\| P - \bar{P} \right\|_v \leq \beta_0 W \qquad (3.12)$$

IV. BEHAVIOUR OF THE MEAN

We shall study the following means

Definition: 1

Markov chain X(t) has the limiting mean $\varphi(t)$ if

$$\lim_{t \to \infty} \left(\varphi(t) - E_k(t) \right) = 0 \qquad (4.1)$$

For any k.

Definition: 2

The double mean of X(t) is defined by

$$E = \lim_{t \to \infty} \frac{1}{t} \int_0^t E_t(u)\, du \qquad (4.2)$$

Provided that limit exists and does not depend on k.

Theorem: Let arrival service and catastrophe rates $\lambda(t), \mu(t), \xi(t)$. Let $\int_0^1 \xi(t) dt > 0.$ Then $X(t)$ has the limiting mean $\varphi(t)$ and double mean E.

Proof.

Given that there exists $\delta > 1$ such that

$$\int_0^1 \left(\xi(t) - (\delta - 1)\lambda(t) \right) dt > 0 \qquad (4.3)$$

Consider a matrix

$$D = diag\,(1, \delta, \delta^2, \ldots) \qquad (4.4)$$

And the space of sequence B such that

$$\left\|X\right\|_B = \sum_{i=0}^{\infty} \delta^i \left|x_i\right| < \infty \quad (4.5)$$

And the forward Kolmagorov system is an equation in the space b. Now we can estimate logarithmic norm $\gamma(A(t))$ in B

$$\gamma(A(t))_B = \sup_{i\geq 0}(\delta\lambda_i(t) - (\lambda_i(t) + \mu_i(t) + \xi(t) + \delta^{-1}\mu_i(t) - (\xi(t) - (\delta-1)\lambda(t)) \quad (4.6)$$

Then

$$\left\|U(t,s)\right\|_B \leq e^{-\int_s^t (\xi(t)-(\delta-1)\lambda(\tau))d\tau} \quad (4.7)$$

For any $0 \leq s \leq t$

Hence $\left\|P^*(t) - p^{**}(t)\right\|_B \leq e^{-\int_s^t (\xi(t)-(\delta-1)\lambda(\tau))d\tau} \left\|P^*(t) - p^{**}(t)\right\|_B$ (4.8)

For any acceptable initial conditions $P^*(t)$, $p^{**}(t)$. On the other hand assumptions of the theorem imply the bound

$$\limsup_{t\to\infty}\left\|P(t)\right\|_B \leq \limsup_{t\to\infty}\left\|U(t)\,P(0)\right\|_B$$

$$+\int_0^t \left\|U(t,\tau)\,g(\tau)d\tau\right\|_B \leq \limsup_{t\to\infty}\int_0^t \xi(t)\,e^{-\int_\tau^t (\xi(t)-(\delta-1)\lambda(u))du} d\tau = M < \infty \quad (4.9)$$

For any P(0)

$$W = \sup_n \frac{n}{\delta^n} < \infty \quad (4.10)$$

Then we obtain

$$\limsup_{t\to\infty} E_{P(0)}(t) = \limsup_{t\to\infty}\sum_{k=0}^{\infty} kp_k(t) \leq W \limsup_{t\to\infty}\left\|P(t)\right\|_B \leq WM \quad (4.11)$$

For any acceptable P(0). Put now $P^*(0) = \pi(0)$, $\phi(t) = \sum_{k=0}^{\infty} k\pi_k(t)$ and $P^{**}(0) = P(0) = e_a$. and then we have in equation (4.9). Then we obtain

$$\left|\varphi(t) - E_0(t)\right| \leq WMe^{-\int_\tau^t (\xi(t)-(\delta-1)\lambda(u))du} \quad (4.12)$$

The right hand side of above equation tends to zero as $t \to \infty$ in accordance with equ (4.11). One can see easily that $\left|\varphi(t) - E_k(t)\right| \to 0$ as $t \to \infty$ for any k. Hence $\varphi(t)$ is limiting mean for X(t). Hence the existence of mean.

V.CONCLUSION

In this paper we have studied Using the stochastic approximation condition: on the single server queuing system with catastrophe and delayed repair. We obtain stability inequalities with an exactcomputation of the constants. We provide explicit analytical expressions for the joint system size probabilities for the state of the background queueing model and the content under steady state differential equation. The probability generating function of the number of customers in the queue is found using the approximation condition. This model can be utilized in large scale manufacturing industries and communication networks.

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