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Solving Fuzzy Linear Programming as Multi Objective Linear Programming Problem

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Abstract **- The constraints and the objective function of the fuzzy linear programming problem are converted into the multi-objective optimization problem (i.e.,) into an equivalent crisp linear problem.Finally, the multi-objective linear programming problem is converted into the weighted linear programming problem to show that they are independent of weights and obtained the complete optimal solution.**

Keywords: **Fuzzy linear programming problem, fuzzy numbers, multi-objective linear programming problem, membership functions, fuzzy co-efficients, tiangular fuzzy number.**

I. INTRODUCTION

In recent times, attempts are made to solve the fuzzy linear programming problems with the help of multi-objective linear programming problem with fuzzy quantities and to show that they are independent of weights**.** This dissertation concerns the solution of fuzzy liner programming (FLP) problem which involves fuzzy numbers both the coefficient matrix of the constraints and the cost coefficients. Then, by using a theorem, the constraints and the objective function of the fuzzy linear programming problem are converted into the multi-objective optimization problem (i.e.,) into an equivalent crisp linear problem. Finally, the multi-objective linear programming problem is converted into the weighted linear programming problem to show that they are independent of weights and obtained the complete optimal solution.

II. LITERATURE SURVEY

E.Bellman and L.A.Zadeh, had already discussed about the "Decision-making in a fuzzy environment", in management science ,Vol. 17, No. 4, December [1]. Guangquan Zhang, Yong-HongWu, M. Remias, Jie Lu, had briefly told about the "Formulation of fuzzy linear programming problems as four-objective constrained optimization problems", in the Department of Mathematics and Statistics, Curtin University of Technology, G.P.O. BoxU1987, Perth, WA 6845, Australia [2]. H.J.Zimmermann, proposed the "Fuzzy programming and linear programming with several objective functions", *in* (operations research), (1978) 45-55 [14].Rafail *N.* Gasimov, *K¨ur_sat Yenilmez*, has given a literature survey on "Solving Fuzzy Linear Programming Problems with Linear Membership Functions", Turk J Math 26 (2002), 375-396 [10]

III. PROBLEM FORMULATION

Linear programming is an algebraic method used to solve sets of linear equations. The formal methodology was developed around 1947. The purpose of linear programming is to find optimal solutions for systems which are modeled by linear equations. In LP, sharp constraints combine to limit the space of feasible solutions to the linear problem being posed. The variable dimensions of the system being modeled assume the form of a vector. The objectives of a problem are also modeled with linear equations. The linearity of the constraints and the objectives enables straight-forward solution methods. Vertices of the solution space correspond to optimizing vectors. The vectors are optimizing in the sense that non-zero linear equations of the system variables, representing the objectives, achieve maximal values at the vertices of the feasible solution space.

Consider a fuzzy linear programming problem where both the coefficient matrix of the constraints and cost coefficient are fuzzy in nature.

$$\langle \tilde{c}, x \rangle = f_i(x_j) = f_i(x) = Max\tilde{Z} = \sum_{j=1}^{n} \tilde{c}_j x_j \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Subject to

$$\sum_{j=1}^{n} \tilde{A}_{ij} x_j \le \tilde{B}_i \ ; \ 1 \le i \le m, \text{ there exists } x_j > 0$$

Then the linear membership function for the above FLPP is,

$$\mu a_{ij}(x) = \begin{cases} 1 & \text{for } x < a_{ii} \\ \dfrac{(a_{ij} + d_{ij} - x)}{d_{ij}} & \text{for } a_{ij} \le x \le a_{ij} + d_{ij} \\ 0 & \text{for } x \ge a_{ii} + d_{ii} \end{cases}$$

$$\mu_{B_i}(x) = \begin{cases} 1 & \text{for } x < b_{ii} \\ \dfrac{(b_i + p_i - x)}{p_i} & \text{for } b_i \le x \le b_i + p_i \\ 0 & \text{for } b_i + p_i \le x \end{cases}$$

Let A be a triangular fuzzy number represented by three crisp numbers s, l, r.
Therefore,

(14) becomes,

$$\langle \tilde{c}, x \rangle = f_i(x_j) = Max \sum_{j=1}^{n} \tilde{c}_j x_j$$

such that,

$$\sum_{x \ge 0} (s_{ij}, l_{ij}, r_{ij}) x_{ij} \le (t_i, u_i, v_i)$$

$$0 \le i \le m, \; 0 \le j \le n$$

where,

$A_{ij} = \langle s_{ij}, l_{ij}, r_{ij} \rangle$, $B_{ij} = \langle t_i, u_i, v_i \rangle$ are fuzzy numbers

Theorem 3.1

For any two triangular fuzzy numbers $A = \langle s_1, l_1, r_1 \rangle$ and $B = \langle s_2, l_2, r_2 \rangle$, $A \le B$ if and only if $s_1 \le s_2$, $s_1 - l_1 \le s_2 - l_2$, $s_1 + r_1 \le s_2 + r_2$.

Using the above statement fuzzy linear programming problem can be rewritten as,

$$\langle \tilde{c}, x \rangle = f_i(x_j) = Max \sum_{j=1}^{n} \tilde{c}_j x_j$$

Such that,

$$\sum_{j=1}^{n} s_{ij} x_j \le t_i , \; \sum_{j=1}^{n} (s_{ij} - l_{ij}) x_j \le t_i - u_i , \; \sum_{j=1}^{n} (s_{ij} + r_{ij}) x_j \le t_i + v_i , \; x_i \ge 0 \qquad (2)$$

Where the membership functions of $\tilde{c}_j(x)$ is,

$$\mu_{\tilde{c}_j}(x) = \begin{cases} 0 & x < \alpha_i \\ \dfrac{x - \alpha_j}{\beta_j - \alpha_j} & \alpha_i \le x \le \beta_i \\ 1 & \beta_i \le x \le \gamma_i \\ \dfrac{\eta_j - x}{\eta_j - y_j} & \gamma_j < x \le \eta_j \\ 0 & \eta_i < x \end{cases}$$

IV.FUZZY LINEAR PROGRAMMING PROBLEM AS MULTI OBJECTIVE CONSTRAINT OPTIMISATION PROBLEM WITH FUZZY COEFFICIENTS

The multi objective linear programming problem with fuzzy coefficients can be formulated as,

$$\underset{x\in X}{Max}\left\{ f_1(x), f_2(x),..., f_k(x)\right\}$$

Subject to ,

$$\sum_{j=1}^{n} s_{ij}x_j \le t_i\ ,\ \sum_{j=1}^{n}(s_{ij}-l_{ij})x_j \le t_i - u_i\ ,\ \sum_{j=1}^{n}(s_{ij}+r_{ij})x_j \le t_i + v_i\ ,\ x_i \ge 0$$

Where $f_i : R^n \to R^i$

Where R be the set of all real numbers and R^n be an n-dimensional Euclidean space.

Associated with the Multi objective linear programming program, we consider the following weighted linear programming problem.

$$\underset{x\in X}{Max}\left\{ w_1 f_1(x), w_2 f_2(x),..., w_k f_k(x)\right\}$$

(i.e) $\underset{x\in X}{Max}\sum_{m=1}^{k} w_m f_m(x)$

Subject to

$$\sum_{j=1}^{n} s_{ij}x_j \le t_i\ ,\ \sum_{j=1}^{n}(s_{ij}-l_{ij})x_j \le t_i - u_i\ ,\ \sum_{j=1}^{n}(s_{ij}+r_{ij})x_j \le t_i + v_i\ ,\ x_i \ge 0$$

EXAMPLE

Solve the following fuzzy linear programming problem

$$Max\ \ f_i(x_1, x_2) = \tilde{c}_1 x_1 + \tilde{c}_2 x_2$$

Subject to the constraints

$$(3,2,1)\, x_1 + (6,4,1)\, x_2 \le (13,5,2)$$

$$(4,1,2)\, x_1 + (6,5,4)\, x_2 \le (7,4,2)$$

where the membership function of $\tilde{c}_1$ and $\tilde{c}_2$ are

$$\mu_{\tilde{c}1}(x) = \begin{cases} 0 & ,\ x < 7 \\ x-7 & ,\ 7 \le x \le 10 \\ 1 & ,\ 10 \le x \le 14 \\ \dfrac{25-x}{14} & ,\ 14 < x \le 25 \\ 0 & ,\ 25 < x \end{cases}$$

where the membership function of $\tilde{c}_1$ and $\tilde{c}_2$ are

$$\mu_{\tilde{c}2}(x) = \begin{cases} 0 & ,\ x < 20 \\ x-20 & ,\ 20 \le x \le 25 \\ 1 & ,\ 25 \le x \le 35 \\ \dfrac{40-x}{5} & ,\ 35 < x \le 40 \\ 0 & ,\ 40 < x \end{cases}$$

Solution

Here we solve the FLPP using simplex method.

First the problem is converted into multi objective linear programming problem and then solved.
Using,

$$\sum_{j=1}^{n} s_{ij}x_j \le t_i \,,\; \sum_{j=1}^{n}(s_{ij}-l_{ij})x_j \le t_i - u_i \,,\; \sum_{j=1}^{n}(s_{ij}+r_{ij})x_j \le t_i + v_i \;,\; x_i \ge 0$$

the above fuzzy linear programming problem can be written as,

$$Max\;\; f_i\left(x_1,x_2\right)=\tilde{c}_1x_1+\tilde{c}_2x_2$$

Subject to the constraints,

$$3x_1+6x_2\le 13\,, 4x_1+6x_2\le 7\,, x_1+2x_2\le 8\,, 3x_1+x_2\le 3\,, 4x_1+7x_2\le 15\,,$$

$$6x_1+10x_2\le 9, x_1,x_2\ge 0$$

Multi objective linear programming program is,

$$Max\left(7x_1+20x_2, 10x_1+25x_2, 14x_1+35x_2, 25x_1+40x_2\right)$$

Subject to ,

$$3x_1+6x_2\le 13,\; 4x_1+6x_2\le 7,\; x_1+2x_2\le 8,\; 3x_1+x_2\le 3,\; 4x_1+7x_2\le 15,$$

$6x_1+10x_2\le 9, x_1,x_2\ge 0$ Multi objective linear programming program with weights is,

$$Max\left(w\right)=\left(w_1\left(7x_1+20x_2\right)+w_2\left(10x_1+25\right)+w_3\left(14x_1+35x_2\right)+w_4\left(25x_1+40x_2\right)\right)$$

Subject to

$$3x_1+6x_2\le 13,\; 4x_1+6x_2\le 7,\; x_1+2x_2\le 8,\; 3x_1+x_2\le 3,\; 4x_1+7x_2\le 15,$$

$$6x_1+10x_2\le 9,\; x_1,x_2\ge 0$$

where w_1, w_2, w_3, w_4 are weights.

Simplex method is used to solve the problem for different weights.

Therefore $\left(x_1,x_2\right)=\left(0,0.9\right)$

$$Max\left(w\right)\, f\left(x_1^*,x_2^*\right)= f\left(0,0.9\right)=0.9\tilde{c}_2$$

$$\mu_{f(0,0.9)}\left(x\right)=\begin{cases} 0 & , \; x\le 18 \\ 1\,\dfrac{x-18}{4.5} & , \; 22.5 < x \le 31.5 \\ \dfrac{36-x}{4.5} & , \; 31.5 < x \le 36 \\ 0 & , \; x > 36 \end{cases}$$

V.CONCLUSION

Thus we can conclude that the solution obtained in solving fuzzy linear programming problem with the help of multi objective linear programming program with fuzzy quantities are independent of weights.

The interest in fuzzy set theory has also been visible in industrial engineering. This interest is primarily connected with the use of fuzzy controllers in manufacturing, fuzzy expert systems for special areas of industrial engineering, and virtually all types of fuzzy decision making, as well as fuzzy linear programming. Fuzzy set theory is also becoming important in computer engineering and knowledge engineering. Its role in computer engineering, which primarily involves the design of specialized hardware for fuzzy logic. Its role in knowledge engineering involves knowledge acquisition, knowledge representation, and human machine interaction.

Thus far, some areas of endeavors have been particularly active in exploring the utility of fuzzy set theory and developing many diverse applications. These include information and knowledge-base systems, various area of engineering, and virtually all problem areas of decision making.

REFERENCES

[1] E.Bellman and L.A.Zadeh, "Decision-making in a fuzzy environment",management science ,Vol. 17, No. 4, December, 1970printed in U.S.A.

[2] Guangquan Zhang, Yong-Hong Wu, M. Remias, Jie Lu, "Formulation of fuzzy linear programming problems as four-objective constrained optimization problems", Department ofMathematics and Statistics, Curtin University of Technology, G.P.O. BoxU1987, Perth, WA 6845, Australia.

[3] George.J.Klir,Bo Yuan,"Fuzzy sets and logic:theory and applications", Prentice Hall, Inc.

[4] Hamdy.A.Taha,"Operations research, introduction", 2007 by Pearson Education, Inc.

[5] H.R.Maleki and M.Mashinchi,"Fuzzy number linear programming: a probabilistic approach"**,** *J. Appl. Math.* &*Computing Vol.* **15***(2004), No. 1 - 2, pp. 333 - 341*

[6] S.H. Nasseri," A New Method for Solving Fuzzy Linear Programming by Solving Linear Programming", Applied Mathematical Sciences, Vol. 2, 2008, no. 50, 2473 – 2480.

[7] S.H. Nasseri, E.Ardil, A.Yazdani, and R.Zaefarian," Simplex Method for Solving Linear Programming Problems with Fuzzy Numbers", World Academy of Science, Engineering and Technology 10 2005.

[8] S.H.Nasseri, and E.Ardil, "Simplex Method for Fuzzy Variable Linear Programming Problems", World Academy of Science, Engineering and Technology 8 2005

[9] Pandian M Vasant," Application of multi objective fuzzy linear programming in supply production planning problem", universiti teknologi Malaysia, *jurnal teknologi*, 40(d) jun. 2004: 37–48.

[10] Rafail *N*. Gasimov, *K¨ur_sat Yenilmez*, "Solving Fuzzy Linear Programming Problems withLinear Membership Functions", Turk J Math 26 (2002), 375-396.

[11] P.A. Thakre, D.S. Shelar ,S.P. Thakre, "Solving fuzzy linear programming problem as multi-objective linear programming problem", Proceedings of the World Congress on Engineering 2009 Vol II WCE 2009, July 1 - 3, 2009, London, U.K.

[12] Takashi Hasuike., Hideki Katagiri.., and Hiroaki Ishii,"Multiobjective Random Fuzzy Linear Programming Problems based on the Possibility Maximization Model", Graduate School of Information Science and Technology, OsakaUniversity, Received November 25, 2008; accepted March 2, 2009.

[13] Li Xiaozhong, "A general model for fuzzy linear programming problem with fuzzy variables", Department of computer science, liaocheng university.

[14] H.J.Zimmermann,"Fuzzy programming and linear programming with several objective functions",lehrstuhl fiir unternehmensforschung (operations research), (1978) 45-55.

TABLE I MOLPP FOR VARIOUS WEIGHTS

Sr No.	w_1	w_2	w_3	w_4	$\left(x_1^*, x_2^*\right)$
1	0	1	1	0	(0, 0.9)
2	0	1	0.5	0	(0, 0.9)
3	0.2	0.4	0.5	0.2	(0, 0.9)
4	0.1	0.2	0.3	0.4	(0, 0.9)
5	0	0.3	0	0.4	(0, 0.9)
6	0.2	0.4	0.6	0.8	(0, 0.9)
7	0.5	0	0.5	0	(0, 0.9)
8	0	1	1	1	(0, 0.9)
9	0.2	0	0	0.5	(0, 0.9)
10	0.3	0.1	1	1	(0, 0.9)
11	0.5	0.5	0.5	0.5	(0, 0.9)
12	0	0	0.5	0.5	(0, 0.9)
13	0.2	0.5	0.5	0.5	(0, 0.9)
14	0.1	0.2	0.3	0.4	(0, 0.9)
15	0	0.2	0	0.2	(0, 0.9)