Asian Journal of Science and Applied Technology
ISSN: 2249-0698 (P) Vol.4 No.1, 2015, pp.1-4

DOI: https://doi.org/10.51983/ajsat-2015.4.1.914

On $M_X\alpha\psi_{\widetilde{\langle\mathcal{H}\rangle}}$ in terms of Minimal Structure Spaces

M.Parimala
Department of Mathematics, Bannari Amman Institute of Technology, Sathyamangalam – 638401, Tamil Nadu, India
E-mail: rishwanthpari@gmail.com
(Received 6 December 2014; Revised 24 December 2014; Accepted 21 January 2015; Available online 27 January 2015)

Abstract **- In this paper we introduce the notion of $M_X\alpha\psi$-irresolute and $M_X\alpha\psi\widetilde{\langle\mathcal{H}\rangle}$. Further, we derive some properties of $M_X\alpha\psi\widetilde{\langle\mathcal{H}\rangle}$ and Pasting Lemma for $M_X\alpha\psi$-irresolute functions.**
AMS Subject Classification: 54A05, 68U05, 69U10
Keywords: **$M_X\alpha\psi$-closed sets, $M_X\alpha\psi$-open sets, $M_X\alpha\psi$-continuous maps, $M_X\alpha\psi$-irresolute maps, $M_X\alpha\psi\widetilde{\langle\mathcal{H}\rangle}$.**

I. INTRODUCTION

In 1950, H. Maki et al. [6] introduced the notions of minimal structure and minimal space. Also they introduced the notion of m_X-open set and m_X-closed set and characterize those sets using m_X-cl and m_X-int operators respectively. Further they introduced m-continuous functions [11] and studied some of its basic properties. They achieved many important results compatible by the general topology case. Some other results about minimal spaces can be found in [1, 2, 3, 4, 7, 8, 11, 12].

The notion of $M_X\alpha\psi$-closed set and $M_X\alpha\psi$-continuous function were introduced and studied by M. Parimala [10]. In this paper we introduce $M_X\alpha\psi$-irresolute and $M_X\alpha\psi\widetilde{\langle\mathcal{H}\rangle}$. Further, we obtain some characterizations and properties.

II. PRELIMINARIES

In this section, we introduce the $\mathcal{M}$-structure and define some important subsets associated to the $\mathcal{M}$-structure and the relation between them.

Definition 2.1 [6] Let X be a nonempty set and let $m_X \subseteq P(X)$, where $P(X)$ denote the power set of X. Where m_X is an $\mathcal{M}$-structure (or a minimal structure) on X, if φ and X belong to m_X.

The members of the minimal structure m_X are called m_X-open sets, and the pair (X, m_X) is called an m-space. The complement of m_X-open set is said to be m_X-closed.

Definition 2.2 [6] Let X be a nonempty set and m_X an $\mathcal{M}$-structure on X. For a subset A of X, m_X-closure of A and m_X-interior of A are defined as follows:

$m_X\text{-}cl(A) = \cap\{F : A \subseteq F,\ X - F \in m_X\}$
$m_X\text{-}int(A) = \cup\{F : U \subseteq A,\ U \in m_X\}$

Lemma 2.3 [6] Let X be a nonempty set and m_X an $\mathcal{M}$-structure on X. For subsets A and B of X, the following properties hold:

(a) $m_X\text{-}cl(X - A) = X - m_X\text{-}int(A)$ and $m_X\text{-}int(X - A) = X - m_X\text{-}cl(A)$.
(b) If $X - A \in m_X$, then $m_X\text{-}cl(A) = A$ and if $A \in m_X$ then $m_X\text{-}int(A) = A$ $m_X\text{-}int(A) = A$.
(c) $m_X\text{-}cl(\varphi) = \varphi$, $m_X\text{-}cl(X) = X$, $m_X\text{-}int(\varphi) = \varphi$ and $m_X\text{-}int(X) = X$.
(d) If $A \subseteq B$ then $m_X\text{-}cl(A) \subseteq m_X\text{-}cl(B)$ and $m_X\text{-}int(A) \subseteq m_X\text{-}int(B)$.
(e) $A \subseteq m_X\text{-}cl(A)$ and $m_X\text{-}int(A) \subseteq A$.
(f) $m_X\text{-}cl(m_X\text{-}cl(A)) = m_X\text{-}cl(A)$ and $m_X\text{-}int(m_X\text{-}int(A)) = m_X\text{-}int(A)$.
(g) $m_X\text{-}int(A \cap B) = (m_X\text{-}int(A)) \cap (m_X\text{-}int(B))$ and $(m_X\text{-}int(A)) \cup (m_X\text{-}int(B)) \subseteq m_X\text{-}int(A \cup B)$.
(h) $m_X\text{-}cl(A \cup B) = (m_X\text{-}cl(A)) \cup (m_X\text{-}cl(B))$ and $m_X\text{-}cl(A \cap B) \subseteq (m_X\text{-}cl(A)) \cap (m_X\text{-}cl(B))$.

Lemma 2.4 [11] Let (X, m_X) be an m-space and A a subset of X. Then $x \in m_X\text{-}cl(A)$ if and only if $U \cap A \neq \varphi$ for every $U \in m_X$ containing x.

Definition 2.5 [8] A minimal structure m_X on a nonempty set X is said to have the property $\mathcal{B}$ if the union of any family of subsets belonging to m_X belongs to m_X.

Remark 2.6 [5] A minimal structure m_X with the property $\mathcal{B}$ coincides with a generalized topology on the sense of Lugojan.

Lemma 2.7 [2] Let X be a nonempty set and m_X an $\mathcal{M}$-structure on X satisfying the property $\mathcal{B}$. For a subset A of X, the following property hold:

(a) $A \in m_X$ iff $m_X\text{-}int(A) = A$
(b) $A \in m_X$ iff $m_X\text{-}cl(A) = A$
(c) $m_X\text{-}int(A) \in m_X$ and $m_X\text{-}cl(A) \in m_X$.

Definition 2.8. A subset A of an m-space $(X,\ m_X)$ is called

(a) $m_X\alpha$-open set [10] if $A \subseteq m_Xint\left(m_Xcl\left(m_Xint(A)\right)\right)$ and an $m_X\alpha$-closed set if $m_Xcl\left(m_Xint\left(m_Xcl(A)\right)\right) \subseteq A$.

(b) m_X-pre open set [12] if $A \subseteq m_Xint\left(m_Xcl(A)\right)$ and an m_X- pre closed set if $m_Xcl\left(m_Xint(A)\right) \subseteq A$.

(c) m_X-semi open set [12] if $A \subseteq m_Xcl\left(m_Xint(A)\right)$ and an m_X- semi closed set if $m_Xint\left(m_Xcl(A)\right) \subseteq A$.

Definition 2.8. A subset A of an m-space $(X,\ m_X)$ is called

(a) an m_X-semi generalized-closed [12] (briefly m_X-sg-closed) set if m_X-scl(A) ⊆ U whenever A ⊆ U and U is m_X-semi-open in (X, m_X). The complement of an m_X-sg-closed set is called an m_X-sg-open set.

(b) an m_X- ψ -closed [10] set if m_X-scl(A) ⊆ U whenever A ⊆ U and U is m_X-sg-open in (X, m_X). The complement of an m_X- ψ -closed set is called an mX- ψ -open set.

(c) an m_X- αψ -closed [10] set if m_X-ψcl(A) ⊆ U whenever A ⊆ U and U is m_X- α -open in (X, m_X). The complement of an m_X- αψ -closed set is called an m_X - αψ -open set.

III. PROPERTIES OF $\mathcal{M}_X\alpha\psi\widetilde{(\mathcal{H})}$

Definition 3.1. A function $f:(X,m_X) \to (Y,m_Y)$ is called

(a) $\mathcal{M}_X\alpha\psi$-continuous [10] if $f^{-1}(V)$ is $\mathcal{M}_X\alpha\psi$-closed in (X,m_X) for every m_X-closed set V of (Y,m_Y),

(b) $\mathcal{M}_X\alpha\psi$-irresolute if $f^{-1}(V)$ is $\mathcal{M}_X\alpha\psi$-closed in (X,m_X) for every $\mathcal{M}_X\alpha\psi$-closed set V of (Y,m_Y).

Definition 3.2. A function $f:(X,m_X) \to (Y,m_Y)$ is called $\mathcal{M}_X\alpha\psi$-homeomorphism (briefly$\mathcal{M}_X\alpha\psi\,(\widetilde{\mathcal{H}})$.) if both f and f^{-1} are$\mathcal{M}_X\alpha\psi$-irresolute and f is bijective.

Lemma 3.3. Let $B \subseteq H \subseteq (X,\ m_X)$ and let $m_X\backslash H$ be the relative m-spaces of H.

(a) If B is $\mathcal{M}_X\alpha\psi$-closed in (X,m_X), then B is $\mathcal{M}_X\alpha\psi$-closed relative to H.

(b) If B is $\mathcal{M}_X\alpha\psi$-closed in a subspace $(H,\ m_X\backslash H)$ and if H is clopen in (X,m_X).

Note that a subset H of m-space (X,m_X) is $\mathcal{M}_X\alpha\psi$-clopen if and only if H is open and $\mathcal{M}_X\alpha\psi$-closed. We prepare some notations. Let $f : X \to Y$ be a function and H a subset of (X,m_X). Let $f\backslash H : H \to Y$ be the restriction of f to H. We define a function $r_{H,K(f)}: H \to K$ by $r_{H,K(f)}(x) = f(x)$ for any $x \in H$, where $K = f(H)$. Then, $f\backslash H = j \circ r_{H,K(f)}$ holds, where $j : K \to Y$ is an inclusion.

Theorem 3.4. Let H and K be subset of (X,m_X) and (Y,m_Y) respectively.

(a) If $f:(X,m_X) \to (Y,m_Y)$ is $\mathcal{M}_X\alpha\psi$-irresolute and if H is a $\mathcal{M}_X\alpha\psi$-clopen subset of (X,m_X), then the restriction $f\backslash H : (H,\ m_X\backslash H) \to (Y,\ m_Y)$ is $\mathcal{M}_X\alpha\psi$-irresolute.

(b) Suppose that K is a $\mathcal{M}_X\alpha\psi$-clopen subset of $(Y,\ m_Y)$. A function $K : (X,\ m_X) \to (K,m_Y\backslash K)$ is $\mathcal{M}_X\alpha\psi$-irresolute if and only if $j \circ k:(X,m_X) \to (Y,m_Y)$ is $\mathcal{M}_X\alpha\psi$-irresolute, where $j : (K,m_Y\backslash K) \to (Y,\ m_Y)$ is an inclusion.

(c) If $f:(X,m_X) \to (Y,m_Y)$ is a $\mathcal{M}_X\alpha\psi,\mathcal{H}$ such that $f(H) = K$ and K are $\mathcal{M}_X\alpha\psi$-clopen subset, then $r_{H,K(f)}:(H,m_X\backslash H) \to (K,m_Y\backslash H)$ is also $\mathcal{M}_X\alpha\psi,\mathcal{H}$.

Proof: (a) Let F be a $\mathcal{M}_X\alpha\psi$-closed set of $(Y,\ m_Y)$. Since f is $\mathcal{M}_X\alpha\psi$-irresolute, $(f\backslash H)^{-1} = f^{-1} \cap H$, H is $\mathcal{M}_X\alpha\psi$-closed, $(f\backslash H)^{-1}(F)$ is $\mathcal{M}_X\alpha\psi$-closed in $(H,m_X\backslash H)$ by *Lemma 3.3*(a). Therefore $f\backslash H$ is $\mathcal{M}_X\alpha\psi$-irresolute.

(b) *Necessity:* Let F be a $\mathcal{M}_X\alpha\psi$-closed set of (Y,m_Y). Then $(j \circ k)^{-1}(F) = k^{-1}\left(j^{-1}(F)\right) = k^{-1}(F \cap H)$ is a $\mathcal{M}_X\alpha\psi$-closed in $(K,m_Y\backslash K)$ by *Lemma 3.3*(a). Therefore $j \circ k:(X,m_X) \to (Y,m_Y)$ is $\mathcal{M}_X\alpha\psi$-irresolute.

Sufficiency: Let V be a $\mathcal{M}_X\alpha\psi$-closed set of $(K,m_Y\backslash K)$. By *Lemma* $3.3(b)$, $(j \circ k)^{-1}(V) = k^{-1}\left(j^{-1}(V)\right) = k^{-1}(F \cap V) = k^{-1}(F)$ is $\mathcal{M}_X\alpha\psi$-closed in (X,m_X). Therefore k is $\mathcal{M}_X\alpha\psi$-irresolute.

(c) First, it suffices to prove $r_{H,K(f)}:(H,m_X\backslash H) \to (K,\sigma\backslash K)$ is $\mathcal{M}_X\alpha\psi$-irresolute. Let F be a $\mathcal{M}_X\alpha\psi$-closed subset of (Y,m_Y).
$\left(j \circ r_{H,K(f)}\right)^{-1}(F) = (f\backslash H)^{-1}(F) = (f\backslash H)^{-1}(F \cap K) = f^{-1}(F \cap K) = f^{-1}(F) \cap K$ is $\mathcal{M}_X\alpha\psi$-closed in $(H,m_X\backslash H)$ and hence $j \circ r_{H,K(f)}$ is $\mathcal{M}_X\alpha\psi$-irresolute. By (b) $r_{H,K(f)}$ is $\mathcal{M}_X\alpha\psi$-irresolute.

Next we show that ${r_{H,K(f)}}^{-1}:(K,m_Y\backslash K) \to (H,m_X\backslash H)$ is $\mathcal{M}_X\alpha\psi$-irresolute. Since $\left(r_{H,K(f)}\right)^{-1} = r_{H,K(f^{-1})}$ and since f^{-1} is $\mathcal{M}_X\alpha\psi$-irresolute, then using the first argument above for f^{-1} we have $(r_{H,K(f)})^{-1}$ is $\mathcal{M}_X\alpha\psi$-irresolute. Therefore $r_{H,K(f)}$ is a $\mathcal{M}_X\alpha\psi,\mathcal{H}$. By using *Theorem 3.4*, for a $\mathcal{M}_X\alpha\psi$-clopen subset H of (X, m_X), we have a homomorphism called restriction $(r_H)^\#:\mathcal{M}_X\alpha\psi,\mathcal{H}(X,\ H;m_X) \to \mathcal{M}_X\alpha\psi,\mathcal{H}(H,\ m_X\backslash H)$ as follows: $(r_H)^\#(f) = r_{H,K(f)}$ for any $f \in \mathcal{M}_X\alpha\psi,\mathcal{H}(X,\ H;m_X)$. To prove that $(r_H)^\#$ is onto we prepare the following:

Lemma 3.5. **(PASTING LEMMA FOR $\mathcal{M}_X\alpha\psi$-IRRESOLUTE FUNCTIONS)**

Let (X, m_X) be a m-space such that $X = A \cup B$ where A and B are $\mathcal{M}_X\alpha\psi$-clopen subsets. Let $f: (A,\ m_X\backslash A) \to (Y,\ m_Y)$ and $g: (B,\ m_X\backslash B) \to (Y,\ m_Y)$ be $\mathcal{M}_X\alpha\psi$-irresolute functions such that $f(x) = g(x)$ for every $X \in A \cap B$. Then the combination $f\nabla g(x) = f(x)$ for any $x \in A$ and $f\nabla g(y) = g(y)$ for any $y \in B$.

Proof: Let F be $\mathcal{M}_X\alpha\psi$-closed set of $(Y,\ m_Y)$. Then $(f\nabla g)^{-1}(F) = f^{-1}(F) \cup g^{-1}(F)$, $f^{-1}(F) \in \mathcal{M}_X\alpha\psi C(X, m_X)$ by using Lemma. It follows from Theorem 3.17 [13] that $f^{-1}(F) \cup g^{-1}(F)$ is $\mathcal{M}_X\alpha\psi$-closed in (X, m_X). Therefore $(f\nabla g)^{-1}(F)$ is $\mathcal{M}_X\alpha\psi$-closed in (X, m_X) and hence $f\nabla g$ is $\mathcal{M}_X\alpha\psi$-irresolute.

Theorem 3.6. If H is a $\mathcal{M}_X\alpha\psi$-clopen subset of (X, m_X), then $(r_H)^\#:\mathcal{M}_X\alpha\psi(\widetilde{\mathcal{H}})(X,\ H;m_X) \to \mathcal{M}_X\alpha\psi(\widetilde{\mathcal{H}})(H,\ m_X\backslash H)$ is an onto homomorphism.

Proof. Let $k \in \mathcal{M}_X\alpha\psi(\widetilde{\mathcal{H}})(H,\ m_X\backslash H)$. By *Theorem 3.4*, $J_1 \circ k:(H,\ m_X\backslash H) \to (X, m_X)$ is $\mathcal{M}_X\alpha\psi$-irresolute, where $J_1:(H,\ m_X\backslash H) \to (X, m_X)$ is an inclusion. Similarly it is shown that $J_2 \circ l_{X\backslash H}:(X\backslash H,\ m_X(X\backslash H)) \to (X, m_X)$ is $\mathcal{M}_X\alpha\psi$-irresolute, where $J_2:(X\backslash H,\ m_X(X\backslash H)) \to (X, m_X)$ is an inclusion.

By using *Lemma 3.5*, the combination $(J_1 \circ k)\nabla(J_2 \circ l_{X\backslash H}):(X, m_X) \to (X, m_X)$, say k_1 is $\mathcal{M}_X\alpha\psi$-irresolute. It is easily shown that $k_1(x) = k(x)$ for any $x \in H$ and k_1 is bijective and ${k_1}^{-1} = (J_1 \circ k^{-1})\nabla(J_2 \circ l_{X\backslash H}):(X, m_X) \to (X, m_X)$ is also $\mathcal{M}_X\alpha\psi$-irresolute. Therefore $k_1:(X, m_X) \to (X, m_X)$ is the required $\mathcal{M}_X\alpha\psi,\mathcal{H}$ and $(r_H)^\#(k_1) = k$ holds and hence $(r_H)^\#$ is onto. We define an equivalence relation R on $\mathcal{M}_X\alpha\psi,\mathcal{H}(X,\ H;m_X)$ as follows:

fRh if and only if $f(x) = h(x)$ for any $x \in H$. Let $[f]$ be the equivalence class of f. Let $H = \{f \mid f \in \mathcal{M}_X\alpha\psi(\widetilde{\mathcal{H}})(X,\ H;m_X) \text{and } f(x) = x \text{ for any } x \in H\}$. Then, $H = ker(r_H)^\#$ and this is normal subgroup of $\mathcal{M}_X\alpha\psi(\widetilde{\mathcal{H}})(X,\ H;m_X)$. The factor group of $\mathcal{M}_X\alpha\psi(\widetilde{\mathcal{H}})(X,\ H;m_X)$ by H is $\mathcal{M}_X\alpha\psi(\widetilde{\mathcal{H}})(X,\ H;m_X)\backslash H = \{fH \mid f \in \mathcal{M}_X\alpha\psi(\widetilde{\mathcal{H}})(X,\ H;m_X)\}$, where $fH = \{\mu(f,k) \mid k \in H\} = [f]$.

Since $(rH)^\#$ is onto by *Theorem 3.6*, then the relation between the groups $\mathcal{M}_X\alpha\psi(\widetilde{\mathcal{H}})(H,\ H;m_X)$ is investigated as follows:

Theorem 3.7. If H is $\mathcal{M}_X\alpha\psi$-clopen subset of (X, m_X), then $\mathcal{M}_X\alpha\psi(\widetilde{\mathcal{H}})(H,\ m_X\backslash H)$ is isomorphic to the factor group $\mathcal{M}_X\alpha\psi(\widetilde{\mathcal{H}})(H,\ H;m_X)\backslash H$.

Proof. By Theorem, $(rH)^\#:\mathcal{M}_X\alpha\psi(\widetilde{\mathcal{H}})(H,\ H;m_X) \to \mathcal{M}_X\alpha\psi(\widetilde{\mathcal{H}})(H,\ m_X\backslash H)$ is an onto homomorphism. Thus, we have the required isomorphism, $\mathcal{M}_X\alpha\psi(\widetilde{\mathcal{H}})(H,\ \tau\backslash H) \cong \mathcal{M}_X\alpha\psi(\widetilde{\mathcal{H}})(H,\ H;m_X)\backslash H$.

Theorem 3.8. If $\alpha:(X, m_X) \to (Y, m_Y)$ is a $\mathcal{M}_X\alpha\psi(\widetilde{\mathcal{H}})$ such that $\alpha(H) = K$, then there is an isomorphism, $\alpha^\#:\mathcal{M}_X\alpha\psi(\widetilde{\mathcal{H}})(H,\ H;m_X) \to \mathcal{M}_X\alpha\psi(\widetilde{\mathcal{H}})(H,\ H;m_X)$.

Proof. The isomorphism $\alpha^\#$ is defined by $\alpha^\#(f) = \alpha \circ f \circ \alpha^{-1}$. Let $(X\backslash R,\ m_X\backslash R)$ be the quotient m-space of (X, m_X) by an equivalence relation R on X and let $\pi:(X, m_X) \to (X\backslash R,\ m_X\backslash R)$ be the canonical projection.

Definition 3.9. A space (X, m_X) is called $\mathcal{M}_X\alpha\psi$-connected if X cannot be expressed as the disjoint union of two non-empty $\mathcal{M}_X\alpha\psi$-closed sets.

Definition 3.10. A function $f:(X, m_X) \to (Y, m_Y)$ is called $\mathcal{M}_X\alpha\psi$-closed if $f(F)$ is $\mathcal{M}_X\alpha\psi$-closed in (Y, m_Y) for every closed set F of $(X,\ m_X)$.

Definition 3.11. A space (X, m_X) is called $T^\#\mathcal{M}_X\alpha\psi$-space if every $\mathcal{M}_X\alpha\psi$-closed set is m_X-closed.

Theorem 3.12. Let F be a subset of $(X\backslash R,\ m_X\backslash R)$. If $\pi:(X, m_X) \to (X\backslash R,\ m_X\backslash R)$ is $\mathcal{M}_X\alpha\psi$-closed function and $\pi^{-1}(F)$ is $\mathcal{M}_X\alpha\psi$-closed in $(X,\ m_X)$, then F is $\mathcal{M}_X\alpha\psi$-closed, where $(X,\ m_X)$ is a $T^\#\mathcal{M}_X\alpha\psi$-space.

Proof. Let $\pi^{-1}(F)$ is $\mathcal{M}_X\alpha\psi$-closed in (X, m_X). Then $\pi^{-1}(F)$ is closed in (X, m_X), since (X, m_X) is $T^\#\mathcal{M}_X\alpha\psi$-space. Then $\pi(\pi^{-1}(F)) = F$ is $\mathcal{M}_X\alpha\psi$-closed in $(X\backslash R,\ m_X\backslash R)$, since π is a $\mathcal{M}_X\alpha\psi$-closed map.

Theorem 3.13. If $\pi: (X, m_X) \to (X\backslash R,\ m_X\backslash R)$ is $\mathcal{M}_X\alpha\psi$-continuous and the subset F is $\mathcal{M}_X\alpha\psi$-closed $(X\backslash R,\ m_X\backslash R)$, then $\pi^{-1}(F)$ is $\mathcal{M}_X\alpha\psi$-closed in $(X,\ m_X)$.

Proof. Let F be a closed set in $(X\backslash R,\ m_X\backslash R)$. Then F is $\mathcal{M}_X\alpha\psi$-closed in $(X\backslash R,\ m_X\backslash R)$. Since π is $\mathcal{M}_X\alpha\psi$-continuous, Then $\pi^{-1}(F)$ is $\mathcal{M}_X\alpha\psi$-closed in $(X,\ m_X)$.

Theorem 3.14. If the bijective map $\pi: (X, m_X) \to (X\backslash R,\ m_X\backslash R)$ is $\mathcal{M}_X\alpha\psi$-continuous and (X, m_X) is $\mathcal{M}_X\alpha\psi$-connected, then $(X\backslash R,\ m_X\backslash R)$ is $\mathcal{M}_X\alpha\psi$-connected.

Proof. Suppose that $(X\backslash R,\ m_X\backslash R)$ is not $\mathcal{M}_X\alpha\psi$-connected. Therefore $X\backslash R = A \cup B$, where A and B are $\mathcal{M}_X\alpha\psi$-closed set. Then $\pi^{-1}(A)$ and $\pi^{-1}(B)$ are $\mathcal{M}_X\alpha\psi$-closed in (X, m_X) such that $X = \pi^{-1}(A) \cup \pi^{-1}(B)$. Therefore (X, m_X) is not $\mathcal{M}_X\alpha\psi$-connected. It is a contradiction. Therefore $(X\backslash R,\ m_X\backslash R)$ is $\mathcal{M}_X\alpha\psi$-connected.

REFERENCES

[1] M. Alimohammady, M. Roohi. Linear minimal spaces, to appear.

[2] M. Alimohammady, M. Roohi. Fixed Point in Minimal Spaces, Nonlinear Analysis: Modelling and Control, 2005, Vol. 10, No. 4, 305–314

[3] Csaszar. Generalized topology: generalized continuity, *Acta. Math. Hunger.*, 96, pp. 351–357, 2002.6. S.

[4] R.Devi, K.Balachandran and H.Maki, Semi generalized homeomorphisms and generalized semi homeomorphisms in topological spaces, *Indian J. Pure. Appl. Math*, 26(3)(1995), 271-284.

[5] S. Lugojan. Generalized Topology, *Stud. Cerc. Math.*, 34, pp. 348 360, 1982.

[6] H. Maki, J. Umehara, T. Noiri. Every topological space is pre T1/2, *Mem. Fac. Sci. Kochi Univ. Ser. Math.*, pp. 33–42.

[7] H. Maki, K. C.Rao, On generalizing semi-open sets and preopen sets, *Pure Appl Math Sci* 49(1999) pp.17-29.

[8] T. Noiri. On Λ_m-sets and related spaces, in: *Proceedings of the 8th Meetings on Topolgical Spaces Theory and its Application, August 2003*, pp. 31–41.

[9] E. Ott, C. Grebogi, J.A. Jorke. Controlling Chaos, *Phys. Rev. Lett.*, 64, pp. 1196–1199, 1990.

[10] M. Parimala, Upper and lower weakly m_X- αψ -continuous multifunctions, *J. Appl.Computat Math 2012 1:2* pp. 1-4

[11] V. Popa, T. Noiri. On M-continuous functions, *Anal. Univ. "Dunarea Jos"-Galati, Ser, Mat. Fiz, Mec. Teor. Fasc. II*, 18(23), pp. 31–41, 2000.

[12] E.Rosas, N.Rajesh and C.Carpintero, Some new types of open sets and closed sets in minimal structure-I, *Int. Mat. Forum* 4(44)(2009), 2169-2184.

[13] J. Ruan, Z. Huang. An improved estimation of the fixed point's neighbourhood in controlling discrete chaotic systems, *Commun. Nonlinear Sci. Numer. Simul.*, 3,pp. 193 197, 1998.