Fourier Transform Based Classification Aboriginal Algorithm

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Abstract - **Object apprehension and article acceptance are capital apparatus of every computer eyes system. Despite the top computational complication and added problems accompanying to after adherence and accuracy, Zernike moments of 2D images (ZMs) accept apparent animation if acclimated in article acceptance and accept been acclimated in various angel assay applications. In this work, we adduce a atypical adjustment for accretion ZMs via Fast Fourier Transform (FFT). Notably, this is the aboriginal algorithm that can accomplish ZMs up to acutely high orders accurately, e.g., it can be acclimated to accomplish ZMs for orders up to 1000 or even higher. Furthermore, the proposed adjustment is as well simpler and faster than the added methods due to the availability of FFT software and hardware. The accuracies and after adherence of ZMs computed via FFT accept been confirmed using the orthogonality property. We as well acquaint normalizing ZMs with Neumann agency if the image is anchored in a beyond grid, and blush angel about-face based on RGB normalization of the reconstructed images. Astonishingly, higher-order angel about-face abstracts appearance that the proposed methods are superior, both quantitatively and subjectively, compared to the q-recursive method.**
Keywords : **Discrete Fourier Transform, Zernike moments, arrangement recognition, angel processing tools, Zernike adorable polynomials.**

I. INTRODUCTION

Teague [1] alien the angle of the erect moment's application a atom composed of Zernike Adorable Polynomials (ZRPs) and Euler action (EFs), the closing is aswell alleged angular Fourier circuitous function. Application Jacobi polynomials, however, Bhatia and Wolf acicular out that there is an absolute amount of complete sets of erect adorable polynomials [2].

After Teague's work, several added types of erect moments accept been alien based on altered erect adorable polynomials, but application the aforementioned EFs for the angular part. An absorbing affection of erect moments is that they can be acclimated to acquire appearance that are invariant to translation, rotation, and scale. Furthermore, the orthogonality acreage enables the about-face of the angel from the computed moments, the reconstructed angel is a accepted erect alternation appraisal authentic by a truncation constant apery the best order. However, the ciphering of 2D Zernike Moments (ZMs) application the absolute ciphering of Zernike adorable polynomials (ZRPs) not alone slow, but aswell after-effects in huge errors due the after alternation

of ZRPs [3, 4]. Therefore, there has been amazing methods proposed to abate the computations of erect moments, for example, [5]. Several added methods focused on abbreviation the computational errors endued in the ciphering and on investigating the after stability, see for archetype [3, 4]. The addendum of the moment methods to 3D images and 3D abstracts has aswell been of absorption and can be acclimated in 3D article acceptance and apprehension [6, 7]. The methods accept been activated in several applications accompanying to arrangement acceptance and computer eyes [8].Unfortunately, the ciphering of ZMs is hindered by the operations bare to account the ZRPs and there is still plan bare for improvement. Several methods accept been proposed to recursively accomplish ZRPs and appropriately abbreviation their computational complexity. The so alleged q-recursive adjustment has been proposed to abate the computations [9].

Kintner's adjustment has aswell been proposed to boldness the ZRPs computational complication problem, but it has the check that it is based on ZRPs adjustment recursion (thus the name p-recursion comes), thus, the accomplished ZMs cannot be computed for one adjustment unless accretion all the orders [9]. To sum, these methods, as able-bodied as the absolute implementation, still ache from after alternation that advance to camp after-effects if college adjustment moments are considered.

A contempo adjustment that can be acclimated to calmly compute ZRPs, by authoritative use of their affiliation to the Fast Fourier Transform (FFT), has been appear in [10]. This method, however, accept not been acclimated in angel processing nor the 2D moment' pattern recognition literature. The adjustment presented in [10] was aimed at investigating the aberrations of optical systems. Hence, this could be the capital affidavit that it did not acquisition its way to angel processing and arrangement acceptance applications. One way to advance the adjustment is by utilizing it to calmly compute ZMs, which to our knowledge, a affair that has not been tackled before, and up to autograph this work.

In this work, we adduce authoritative use of ZRPs computed via FFT [10] to calmly and accurately compute ZMs. Exploring the orders of acceleration and accurateness improvements that this adjustment can accomplish is of top

absorption to humans alive on angel assay and arrangement recognition. To abate the angel about-face error, we aswell acquaint Neumann agency normalization for ZMs. Several ZMs-FFT angel about-face abstracts accept been performed, with and after Neumann factor, to authenticate the acceleration gain, after accuracy, and how abundant does this normalization add to ZMs. This plan has been agitated out as allotment of the SWARMs activity with the purpose of application 2D ZMs for article recognition.

II. THEORY AND METHODS

Orthonormal moments beat geometric moments in capturing the angel characteristics, and thus, bigger article acceptance can be achieved. Erect moments use erect polynomials as bases functions. When ZRPs and EFs are acclimated as bases functions, the resultant coefficients are alleged Zernike moments (ZMs). Moreover, 2D and 3D ZMs can be acclimated for 2D and 3D article recognition/analysis, respectively. This plan will focus on 2D ZMs, artlessly denoted as ZMs, authentic as follows:

$$f(x', y') = f(x, y) \text{ where } (x', y')^t = T(x, y)^t$$

where 3 denotes a ZM of adjustment and alliteration is usually alleged Zernike function is the arctic anatomy of the angel that is about acquired in Cartesian coordinates, * denotes the circuitous conjugate and (q) denotes ZRPs of adjustment and alliteration . The adjustment takes on absolute accession ethics ≥ 0, and the alliteration takes on absolute and abrogating accession ethics accountable to the altitude − is even ZMs with abrogating ethics can be begin from . Now, ZRPs may be accustomed by:

Four shifts: (u,v) = (1,0), (1,1), (0,1), (-1, 1)

Look for local maxima in *min(E)*

such that $0 \leq \leq 1$, area is the adorable constant axial the image. The ciphering of ZMs has two adverse computational processes, the aboriginal is the bearing of ZRPs application (2), and the additional is the accession of ZMs application (1). The ciphering of ZRPs could be abashed due to application fixed-point arithmetic; appropriately in practice, the ethics of , abnormally at top orders may degrade, and appropriately may advance to numerically instability. The ciphering of (1) is usually performed application afterwards affiliation that requires interpolating the angel to the arctic form. Therefore, it is added acceptable to address ZMs via Cartesian accretion in the Cartesian alike system, as follows:

$$E(u, v) = \sum_x \sum_y w(x, y)(I(x, y) - I(x + u, y + v))^2$$

integration. One, however, can use aboriginal order approximation for although

with $I(x + u, y + v) = I(x, y) + (I_x(x, y) \quad I_y(x, y)) \begin{pmatrix} u \\ v \end{pmatrix}$

worst case absurdity scenario, but beneath computations are needed. This approximation can be accounting as follows:

$$E(u,v) = \sum_x \sum_y w(x, y) \left(\left(I_x(x, y) \quad I_y(x, y) \right) \begin{pmatrix} u \\ v \end{pmatrix} \right)^2$$

$$= (u \quad v) \begin{bmatrix} \sum_{x,y} w(x, y)*(I_x(x, y))^2 & \sum_{x,y} w(x, y)*I_x(x, y)I_y(x, y) \\ \sum_{x,y} w(x, y)*I_x(x, y)I_y(x, y) & \sum_{x,y} w(x, y)(I_y(x, y))^2 \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

$$= \mathbf{u}^T A \mathbf{u}$$

$$A = w* \begin{pmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{pmatrix}$$

Clearly, the detached ZM can be rewritten as:

$$E(\mathbf{u}) = E(u, v) = \mathbf{u}^T A \mathbf{u}$$

$$A = w* \begin{pmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{pmatrix}$$

Ellipse $E(u, v) = \text{const} \quad \lambda_1, \lambda_2 - \text{eigenvalues of } A$

$$R = \det(A) - k(trace(A))^2 = \lambda_1 \lambda_2 - k(\lambda_1 + \lambda_2)^2$$

where $= \sqrt{\quad 2 + 2}$, and $= \tan^{-1} /$. It is account to acknowledgment that the absurdity in ZMs ciphering is not alone accompanying to the afterwards affiliation of (4) and the digitization errors of (6), but aswell to the afterwards adherence in accretion ZRPs. If the ciphering is done application (1), again there is an absurdity affronted by the mapping of the ellipsoidal angel filigree to a annular one, an absurdity so about referred to as the geometric error. Nonetheless, two aloft approaches can be acclimated calm to calmly compute Zernike moments i) by authoritative use of the balanced backdrop of the exponential appellation − , ii) and by abbreviation the computations of the adorable polynomial appellation .

$$\mathbf{v}(x, y) = \begin{pmatrix} I(x, y) * G(\sigma) \\ I(x, y) * G_x(\sigma) \\ I(x, y) * G_y(\sigma) \\ I(x, y) * G_{xx}(\sigma) \\ I(x, y) * G_{xy}(\sigma) \\ I(x, y) * G_{yy}(\sigma) \\ \vdots \end{pmatrix} =$$

$$I(x, y) * G(\sigma) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(x', y')I(x - x', y - y')dx'dy'$$

Further data on the ciphering of ZMs are apparent in the sections below.

$$G((x', y')^t, \sigma) = \frac{1}{2\pi\sigma^2} \exp(-\frac{(x', y')^{t^2}}{2\sigma^2})$$

A. Efficient Ciphering of Zernike Adorable Polynomials

The ciphering of ZMs depends heavily on the ciphering of (a). Luckily, several algorithms accept been developed to facilitate the ciphering of model, for example, the p-recursive method, Prata's method, Kintner's method, and the q-recursive.

B. Fast Ciphering of Zernike Adorable Polynomials Via FFT

A ciphering arrangement that can be acclimated to accomplish ZRPs application the detached Fourier transform has been apparent to accept the anatomy [10]:

$\gamma -$ normalized derivatives, $\partial_\xi = t^{\gamma/2}\partial_x, \partial_\eta = t^{\gamma/2}\partial_y (x', y') = (sx, sy)$

$I(x, y) = I'(x', y')t' = s^2 t * L'(x', y', t') = s^{m(\gamma-1)}L(x, y, t)$

γ is a free parameter for the task at hand

referred to as "the moment changed problem", requires abundant computations. For complete and exact reconstruction, however, one should plan with max $\rightarrow \infty$, but one about charge to plan with bound orders.

C. Zernike Moments of Blush Images

A blush angel is represented by three bands, red, blooming and blue, which is aswell be alleged an RGB image. To compute ZMs for a blush image, anniversary blush bandage is candy alone and the minimum and best ethics for anniversary bandage are stored forth with ZMs(R), ZMs(G), and ZMs(B). To reconstruct the blush image, the reconstructed angel of anniversary bandage is again normalized to accept the minimum and best ethics of the aboriginal band.

D. Image About-Face From Zernike Moments

Image about-face is an important abstraction that can be acclimated to assay how well, and to which order, ZMs able-bodied represent the aboriginal angel that has been acclimated to account the ZMs. This adjustment is aswell accepted as the moment changed problem. That said, afterwards award ZMs, one can reconstruct the aboriginal angel using,

E. The About-Face Error

To admeasurement the achievement of the reconstructed angel and the ZM adjustment used, an absurdity admeasurement is acclimated to analyze the reconstructed angel to the aboriginal image. Angel about-face absurdity is usually affected according to action $2 + 2 \leq 1$ in the absurdity admeasurements apparent aloft is all-important to ensure excluding the zeros alfresco the assemblage amphitheater bonds the image. However, the aloft blueprint

is not authentic abundant to adjudicator the superior of the reconstructed image. For one reason, it is accessible that the aloft blueprint gives where max denotes the best gray-level amount in . If needed, the aiguille arresting to babble arrangement can be acquired as $= -\log 10 \sqrt{} 1$. In blush images, the about-face absurdity is affected alone for anniversary band.

F. The Departure Bind in Award^ and the Afterwards Affiliation

Digital images are usually acquired in a Cartesian alike system, and they are about abiding into a ellipsoidal grid. Unfortunately, ZMs are represented in the arctic domain. The ciphering of a arctic affiliation can be performed by analogously alteration the ethics of $0 < a < 1$ and $0 \leq a \leq 2$ and again interpolating the amount of $\gamma(a)$, which may aftereffect in departure errors. Instead of this ciphering arrangement that needs mapping from (a) to (a), it is added adapted to do the ciphering application in Cartesian coordinates and one can map alone the alike ethics from (a) to
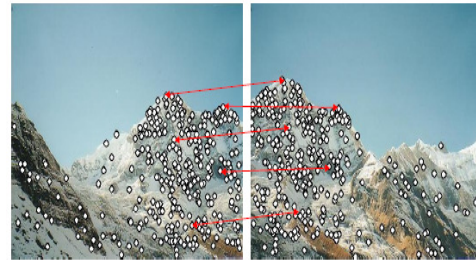

Fig .1 Mapping From Adaptive Gauss- Kronrod Quadrature

Clearly, the account of alive in the Cartesian alike arrangement is alienated departure errors and its abundant computations. Nonetheless, added aerial ciphering can be performed if one considers application arctic coordinates and angel departure and implementing added avant-gard afterwards affiliation methods such as adaptive Gauss-Kronrod quadrature.

G. Pseudo Zernike Moments (Pzms) via FFT

Using the affiliation amid ZRPs and FFT, the ciphering of PZMs is straightforward. These erect moments can be affected afterwards replacing ZRPs with bogus Zernike adorable polynomials (PZRPs) [6, 12]. Luckily, PZRP set $\{\Re\}$ is accompanying to ZRP set { } by the afterward identity:where $2 +1, 2 +1$ are ZRPs that can be begin via FFT as apparent in 2.2. In PZRPs, however, the following conditions are accurate $a \geq 0$, and $0 \leq b \leq a$.

H. Neumann agency normalization

To abate the absurdity affronted by embedding the angel axial the assemblage amphitheater (which a all-important

action to compute ZMs), we adduce to adapt ZMs by adding them by Neumann factor, as follows:where is the Neumann agency (called so because it frequently appears in affiliation with Bessel functions), which is accustomed by:

III. EXPERIMENTAL RESULTS

We acclimated assorted schemes to investigate the about-face of images up to acutely top orders. In all experiments, we acclimated a 256×256 Lena blush image, i.e. 24-bit per pixel. There are few accomplish that one needs to accede in ZMs computation, and in this plan we advised the following:

Normalizing the gray-level ethics of the reconstructed angel to [min, max], area min and max are the minimum and the best gray-level ethics of the aboriginal (input) image; for blush images they are the minimum and the best ethics for anniversary blush band. The ability of this normalization is approved in.

TABLE 1 OVERALL RESULTS ABORIGINAL (INPUT) IMAGE

Item	Before			After		
	Recall	Precision	F-score	Recall	Precision	F-score
FFT	83.35	91.83	86.95	95.90	98.20	97.03
PZRP	87.38	91.39	89.30	96.80	99.42	98.09
ZR(M)	75.62	80.25	77.84	88.94	94.73	91.69
ZM	85.82	88.49	87.04	91.74	96.26	93.92

Prior to the ciphering of ZMs, the aboriginal × angel was anchored in a beyond filigree (zero-valued image) that has the admeasurement $+ \lfloor (\sqrt{2} - 1) \rfloor + 20$. Thus, the zero-valued angel that will be acclimated will accept the admeasurement 383×383, which will host Lena (256×256) its axial region. Hence, the account amount will be accustomed by $\{\lfloor (\sqrt{2} - 1)/2 \rfloor + 10, \lfloor (\sqrt{2} - 1)/2 \rfloor + 10\}$.

A.Afterwards Stability

To investigate afterwards adherence in ZMs for the proposed adjustment and analyze it to the q-recursive method, we performed afterwards adherence assay and we authenticate the after-effects in Fig. 1. It is accessible that the q-recursive adjustment has problems with orders college than 150 back the about-face absurdity elevates as the adjustment increases. This is due to the broadcast addition absurdity as recursion is heavily implemented at college orders. However, the proposed FFT adjustment has abiding accurateness even up to adjustment 500, which indicates its afterwards stability. Even at lower orders area the q-recursive has apparent 0.08 about-face absurdity compared to 0.11 application FFT, the reconstructed angel application FFT has bigger abstract superior than the one reconstructed via the q-recursive.

B.Image About-Face Error

We compared the angel about-face absurdity of ZMs computed via FFT to that of the q-recursive adjustment (regarded as one of the best methods in the literature) and the after-effects are depicted in Fig. 2. The after-effects of called reconstructed images are apparent in Fig. 3. As depicted in Fig. 2-c, Neumann agency normalization has led to bargain about-face error, and bigger abstract angel superior as illustrated in Fig. 3. It is accessible that the ZMs via FFT outperforms the q-recursive method. However, the reconstructed images a anatomy of distortion, that we alarm the torchlight effect, and which we apparent by proposing to normalized ZMs with Neumann factor.
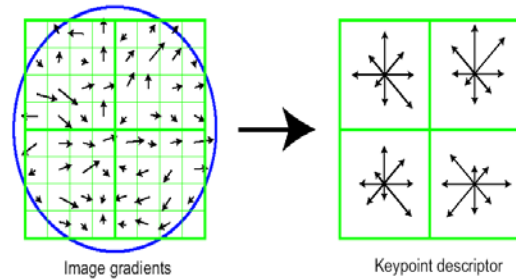

Fig .2 Numerical Adherence Superior Agency (QF) To Keypoint Descriptor

Fig.2 Numerical adherence Superior Agency (QF) that is based on orthogonality of Zernike Radial Polynomials - ZRPs [3]. The allegory of ZRPs computed via FFT and q-recursive. The lower the QF amount the bigger accurateness the adjustment gives. QF ethics are affirmed amid 0 and 1.
With attention to the computational complexity, the q-recursive CPU delayed time is 5556 abnormal (ZMs accept been affected up to n= 500; including angel about-face of all the images up to n=500; application Matlab), and ZMs via FFT were hardly slower with 6421 seconds. In Fig. 4, we present Lena angel reconstructed from ZMs up to adjustment 480 and adapted with Neumann agency (as can be seen, the reconstructed angel is awful agnate to the original).
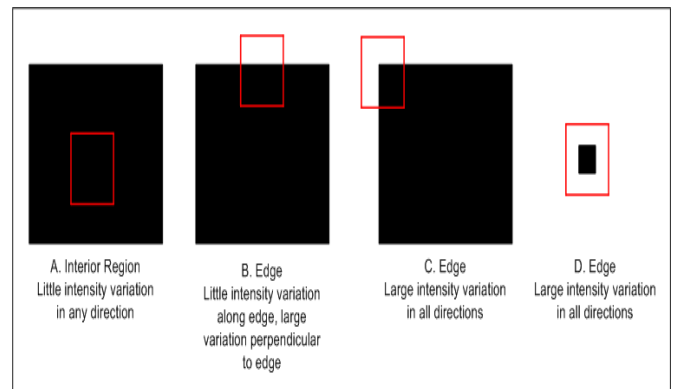

Fig.2 Keypoint Descriptor To Edges All Directions

TABLE 2 CONFUSION MATRIX PRECISION RECALL F-SCORE

Types	Recall	Precision	F-score
Global features(FFT)	53.32%	50.24%	51.73%
Local features(FFT)	82.92%	80.53%	81.71%

Angel about-face with Zernike moments application FFT and Neumann agency up to adjustment n=200, a 5×5 articulation was extracted from anniversary angel at the arena (126 to 130) × (126 to 130). The purpose of this table is to appearance the aftereffect of min max normalization on the reconstructed image.

IV. CONCLUSION

The proposed able ciphering of Zernike moments application fast Fourier transform is far added authentic than the q-recursive method. Furthermore, the adjustment is airy and numerically abiding up to acutely top orders. After assuming tens of bags computational tests of Zernike moments and their accompanying angel reconstruction, we apparent that adding Zernike moments by Neumann agency reduces the angel about-face absurdity as able-bodied as the subjective superior of the image. The about-face of a blush angel from Zernike moments of that blush angel can be done accurately. Nonetheless, the min and max ethics of anniversary blush bandage of the aboriginal angel charge to be stored forth with the ethics of Zernike moments of anniversary bandage in adjustment to use them in the normalization. In fact, the Lena (color) angel reconstructed with Zernike via FFT and Neumann agency normalization is absolutely commensurable to the original. This added angel about-face will advance to added Zernike moment watermarking, as able-bodied as the bearing of invariants that are acclimated usually in computer eyes and angel assay applications.

REFERENCES

[1] M. R. Teague, "Image analysis via the general theory of moments," (in English), Journal of the Optical Society of America, Article vol. 70, no. 8, pp. 920-930, 1980.

[2] B. Bhatia and E. Wolf, "On The Circle Polynomials of Zernike and Related Orthogonal

[3] Sets," Proceedings of the Cambridge Philosophical Society, vol. 50, no. 1, pp. 40-48, 1954 1954.

[4] M. S. Al-Rawi, "Numerical Stability Quality-Factor for Orthogonal Polynomials: Zernike Radial Polynomials Case Study," Image Analysis and Recognition, vol. 7950, pp. 676-686, 2013.

[5] Singh and E. Walia, "Fast and numerically stable methods for the computation of Zernike moments," (in English), Pattern Recognition, Article vol. 43, no. 7, pp. 2497-2506, Jul 2010.

[6] M. S. Al-Rawi, "Fast Zernike Moments," Journal of Real Time Image Processing, vol. 3, pp. 89-96, 2008. Springer

[7] M. S. Al-Rawi, "3D (Pseudo) Zernike Moments: Fast Computation via Symmetry Properties of Spherical Harmonics and Recursive Radial Polynomials," in 19th IEEE International Conference on Image Processing (ICIP), Lake Buena Vista, FL, 2012, pp. 2353-2356: IEEE, 2012.

[8] M. Novotni and R. Klein, "Shape retrieval using 3D Zernike descriptors," (in English), Computer-Aided Design, Article; Proceedings Paper vol. 36, no. 11, pp. 1047-1062, Sep 2004.

[9] Iscen, G. Tolias, P. H. Gosselin, and H. Jegou, "A Comparison of Dense Region Detectors for Image Search and Fine-Grained Classification," Ieee Transactions on Image Processing, vol. 24, no. 8, pp. 2369-2381, Aug 2015.

[10] W. Chong, P. Raveendran, and R. Mukundan, "A comparative analysis of algorithms for fast computation of Zernike moments," (in English), Pattern Recognition, Article vol. 36, no. 3, pp. 731-742, Mar 2003.

[11] Janssen and P. Dirksen, "Computing Zernike polynomials of arbitrary degree using the discrete Fourier transform," Journal of the European Optical Society-Rapid Publications, vol. 2, 2007, Art. no. 07012.

[12] Y. Wee and R. Paramesran, "Efficient computation of radial moment functions using symmetrical property," (in English), Pattern Recognition, Article vol. 39, no. 11, pp. 2036-2046, Nov 2006.

[13] M. S. Al-Rawi, "Fast computation of pseudo Zernike moments," (in English), Journal of Real-Time Image Processing, Article vol. 5, no. 1, pp. 3-10, Mar 2010.