Asian Journal of Science and Applied Technology
ISSN: 2249-0698 (P) Vol.6, No.2, 2017, pp.1-7

DOI: https://doi.org/10.51983/ajsat-2017.6.2.992

Endogeneity Violation on the Comparison of Ordinary Least Square and Maximum Likelihood Extraction Method of Factor Analysis

Alabi Oluwapelumi and O.J.Kayode
Department of Mathematics and Statistics Rufus Giwa Polytechnic,
P. M. B. 1019, Owo, Ondo State, Nigeria
E-Mail: oluwapelumialabi@yahoo.com

(Received 11 May 2017; Accepted 23 June 2017; Available online 1 July 2017)

Abstract - **One of the main objectives of factor analysis is to reduce the number of parameters. The number of parameters in the original model is equal to the number of unique elements in the covariance matrix. The study compared ordinary least square and maximum likelihood method of extraction of factor analysis under two approaches such that the variables employed were assumed to be independent of error i.e endogeneity assumption in the first approach while the endogeneity assumption is violated by omitting the important variable HLT in the second approach. The result showed that the extracted factors under the violation of endogeneity has similar factors loading pattern which accounted for a great deal of variance and the factors do a good job of representing the original data and the Bayesian information criterion also showed that the maximum likelihood method of extraction slightly outperforms ordinary least square.**

Keywords: **Factor analysis, Ordinary least square, Maximum likelihood, Endogeneity assumption.**

I. INTRODUCTION

Factor analysis operates on the notion that measurable and observable variables can be reduced to fewer latent variables that share a common variance and are unobservable, which is known as reducing dimensionality. These unobservable factors are not directly measured but are essentially hypothetical constructs that are used to represent variables. For example, scores on an oral presentation and an interview exam could be placed under a factor called ‘communication ability’; in this case, the latter can be inferred from the former but is not directly measured itself. Exploratory factor analysis is used when a researcher wants to discover the number of factors influencing variables and to analyze which variables ‘go together’. A basic hypothesis of exploratory factor analysis is that there are common ‘latent’ factors to be discovered in the dataset, and the goal is to find the smallest number of common factors that will account for the correlations (one of the main objectives of factor analysis is to reduce the number of parameters. The number of parameters in the original model is equal to the number of unique elements in the covariance matrix).

Factor analytic methods trace their history to Spearman's (1904) seminal article on the structure of intelligence, and were eagerly adopted and further developed by other intelligence theorists (e.g. Thurstone, 1936). In factor analysis there are various techniques or methods of extracting factors, in this study we considered ordinary least squares (minimum residual) and maximum likelihood factor analysis under the assumption of endogeniety (i.e exogeneous independent of error term) in ordinary least squares.

The primary goal of ordinary least squares (OLS) method for obtaining factor solutions is to minimize the sum of squared differences between the observed and implied covariance matrices. The OLS method for extracting factors assigns weights to residuals of large and small factors equally; unlike maximum likelihood methods for extracting factors. Based on the assumption that a specified number of factors exists in a population, maximum likelihood factor analysis yields estimates of factor loadings for a given sample size and number of observed variables. When the observed variables exhibit multivariate normality and the sample size is large, maximum likelihood strategies facilitate the calculation of confidence intervals for the estimated loadings.

Based on the assumption that a specified number of factors exists in a population, maximum likelihood factor analysis yields estimates of factor loadings for a given sample size and number of observed variables. When the observed variables exhibit multivariate normality and the sample size is large, maximum likelihood strategies facilitate the calculation of confidence intervals for the estimated loadings.

An endogeneity problem occurs when an explanatory variable is correlated with the error term. This implies that the regression coefficient in an ordinary least squares regression is biased, however if the correlation is not contemporaneous, then it may still be consistent. There are many methods of overcoming this, including instrumental variable regression and Heckman selection correlation. The following are the common sources of endogeneity, omitted-variable, measurement error and simultaneity. For the purpose of this study, omitted variable would be considered. In the case of omitted variable, the homogeneity come from uncontrolled confounding variable. A variable is both correlated with an independent variable in the model and with the error term. Equivalently, the omitted variable both affects the independent variable and separately affects the dependent variable.

Several works have been done in factor analysis. So far, different extracting methods of factor analysis such as maximum likelihood, ordinary least squares have been compared in one way or the other improve the quality of evaluative research in civics education, Finkel and Ernst (2005) presented the findings of a study conducted in 1998. The study examined the "impact of civic education on South African high school students. Finkel et al (2005) used a battery of items to determine students' "political knowledge, civic duty, tolerance, institutional trust, civic skills, and approval of legal forms of political participation" and the observed variables under respective items were measure on different scales such as: binary, ordinal, and interval scales. Their results were presented through a table containing two sets of factor loading coefficients; one set of coefficients were associated with students who received civics education, and another set was associated with students who received no civics education. Through comparing the strengths of loading coefficients, the researchers highlighted slight differences in loadings between the groups.

Vanzile-Tamsen *et al.* (2006), employed a maximum likelihood confirmatory factor analysis to compare three models of risk behavior. The comparisons were based on three fit indices, based on the results of these comparisons, the authors proposed a four latent factor model to account for their observations.

Winter *et al* (2011) conducted simulations to investigate factor recovery by principal axis factoring and maximum likelihood factor analysis for distortions of ideal simple structure and sample sizes between 25 and 5000. Results showed that principal axis factoring is preferred for population solutions with few indicators per factor and for over extraction.

Coughlin (2013) employed Monte Carlo method to simulate data under 540 different conditions; specifically, the study is a four (sample size) by three (number of variables) by three (initial communality levels) by three (number of common factors) by five (ratios of categorical to continuous variables) design. Factor loading matrices derived through the tested factor extraction methods were evaluated through four measures of factor pattern agreement and three measures of congruence. Across the majority of interactions among the manipulated research contexts that accounted for statistically significant differences and moderate effect sizes, the ordinary least squares factor extraction method yielded factor loading matrices that were in better agreement with the population than either the maximum likelihood or the principal axis methods, also yielded factor loading matrices that exhibited less bias and error than the other two tested factor extraction methods. In general, ordinary least squares loading matrices resulted in factor scores that correlated more strongly with population factor scores than the other tested methods.

Zygmont and Smith (2014) discusses robust analytical alternatives for answering nine important questions in exploratory factor analysis (EFA), and provides R commands for running complex analysis in the hope of encouraging and empowering substantive researchers on a journey of discovery towards more knowledgeable and judicious use of robust alternatives in factor analysis. However no existing studies have addressed any method of extraction of factor analysis under the assumption of endogeneity. This study will differ from existing studies by employing ordinary least square and maximum likelihood method of extraction of factor analysis under the assumption of endogeneity of least squares.

II. MATERIALS AND METHODS

The study is based on time series data obtained from National Bureau of Statistics bulletin Republic of Nigeria that covers the period 31 years from 1981 to 2011. The data consist of GDP, Agriculture, Crude Oil and Gas, Building and Construction, wholesale and Retail, Telecommunication, Financial Institutions, Education and Health sectors.

1.Endogeneity

This occurs when an explanatory variable is correlated with the error term. This implies that the regression coefficient in an ordinary least squares regression is biased, however if the correlation is not contemporaneous, then it may still be consistent.
For the purpose of this study we considered omitted variable.
Assume that the "true" model to be estimated is

$$y_i = \alpha + \beta x_i + \gamma z_i + u_i$$

but we omit (perhaps because we don't have a measure for it) when we run our regression. It will get absorbed by the error term and we will actually estimate,

$y_i = \alpha + \beta x_i + \varepsilon_i$, where $\varepsilon_i = \gamma z_i + u_i$ If $\gamma \neq 0$, then is correlated with the error term.

2.Ordinary Least Squares Factor Analysis

In Ordinary Least Squares factor analysis, the relationship between factor pattern matrices, *A*, and implied correlation matrices, $\hat{R}$, is given by (Cureton & D'Agostino,1983; Harman, 1976): $\hat{R} = AA'$ The least squares solution can be found by "fitting $(R - I)$ by $(\hat{R} - H^2)$" (Harman, 1976, p. 176).
where $H^2 = I - U^2 = diag(AA')$ *

The diagonal matrix described in equation (*) contains communalities. Minimizing the off-diagonal residuals results in the ordinary least squares method for developing factor solutions (Cureton & D'Agostino, 1983; Harman, 1976). This minimization is given by the following expression (Harman, 1976):

$$\min_A \left[[R-I]-[AA'-diag(AA')]\right]$$

The function to be minimized can be written algebraically as:

$$f(A)=\sum_{k=j+1}^{n}\sum_{j=1}^{n-1}\left(r_{jk}-\sum_{p=1}^{m}a_{jp}a_{kp}\right)^2$$

Through varying the values of factor loadings, the objective is to minimize this function for a specified number of factors, *m* (Harman, 1976). To develop a function that is independent of the number of variables in the sample correlation matrix, Harman (1976) suggests the minimization of the root-mean-square deviation (*rms*); this is given by:

$$\text{RMS}=\sqrt{\frac{2f(a)}{n(n-1)}}$$

In addition to ensuring that the fitting function is independent of the order of the correlation matrix, the ordinary least squares method requires communality estimates to be less than or equal to one; communality estimates are restricted to values between zero and one via the following condition (Harman, 1976):

$$h_j^2=\sum_{p=1}^{m}a_{jp}^2\leq 1$$

The iterative process through which $f(A)$ is minimized involves small changes in the variables, and the resulting variables replace the original ones (Harman, 1976). Specifically, "for any row *j* in *A* an increment $\varepsilon_p(p=1,2\ldots,\text{m})$ is added to each element:

$a_{ji}+\varepsilon_1+a_{j2}+\varepsilon_2,\ldots a_{jp}+\varepsilon_p,\ldots,a_{jm}+\varepsilon_m$ " (Harman, 1976, pp. 177-178). The new factor loadings are described in the following form:

$$b_{jp}=a_{jp}+\varepsilon_p \qquad (j=1,2,\ldots,\text{m})$$

The reproduced correlation matrix of a given variable j with any other variable k
is given by (Harman, 1976):

$$\hat{r}_{jk}=\sum_{p=1}^{m}a_{kp}b_{jp}$$

The sum of squared residuals correlations is (Harman, 1976):

$$f_j=\sum_{\substack{k=j\\k\neq j}}^{n}\left(r_{jk}=\sum_{p=1}^{m}a_{kp}b_{jp}\right)^2$$

When the original factor loading is "separated from the incremental change," the above
Equation becomes (Harman, 1976, p. 178):

$$f_j=\sum_{\substack{k=j\\k\neq j}}^{n}\left(r_{jk}^*=\sum_{p=1}^{m}a_{kp}\varepsilon_p\right)^2$$

With the incremental changes in factor loadings removed, the original residual correlations, r_{jk}^*, of variables k with a fixed j are given by (Harman, 1976):

$$r_{jk}^*=r_{jk}-\sum_{p=1}^{m}a_{kp}a_{jp} \quad (\text{k}=1,2,\ldots,\text{n};\ \text{k}\neq\text{j})$$

The first step in determining the values of □ that minimize the function □ involves taking the partial derivatives of the sum of squared residual correlations with the original factor loadings separated from the incremental changes; this expression
becomes:

$$\frac{\partial f_j}{\partial_{\varepsilon q}}=2\sum_{\substack{k=j\\k\neq j}}^{m}\left(r_{jk}^*=\sum_{p=1}^{m}a_{kp}\varepsilon_p\right)(-a_{kq})(q=1,2,\ldots,m)$$

In the second step, these equations are set to zero; this leads to the following "implicit equations" (Harman, 1976, p. 178):

$$\sum_{p=1}^{m}\left(\sum_{\substack{k=1\\k\neq j}}^{n}a_{kp}a_{kq}\right)\varepsilon_p=\sum_{\substack{k=1\\k\neq j}}^{m}r_{jk}^*a_{kq}(q=1,2,\ldots,\text{m})$$

$$\varepsilon_j A'_{)j(}A_{)j(}=r_j^0A(\varepsilon_j=\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_m)$$

In this expression,

a) ε_j is a row vector of incremental changes of the factor loadings for variable j;

b) $A_{)j(}$ is the factor loading matrix with the elements in row j replaced with zeros;

c) r_j^0 is the row vector of residual correlations of j with all other variables.

The solution for the values of ε_j that minimize the function f is given by (Harman,1976): $\varepsilon_j=r_j^0A(A'_{)j(}A_{)j(})^{-1}$

In ordinary least squares, Heywood cases, or factor solutions that imply communalities greater than one, are not considered proper solutions. Therefore, when solutions that minimize the function f result in communalities that are greater than one, the following constraint is applied (Harman, 1976): $\sum_{p=1}^{m}b_{jp}^2\leq 1$

This constraint ensures that minimum values of the fitting function yield proper factor solutions.

3. Maximum Likelihood Factor Analysis

Before using a maximum likelihood strategy to develop estimates of common factor loadings, a researcher must first use sample data to create a distribution of the elements of a covariance matrix. When samples of observations are drawn from multivariate normal distributions, the distribution function of the elements of the covariance matrix can be defined as (Harman, 1976):

$$dF = K\left|\sum\right|^{-\frac{1}{2}(N-n-2)} \exp -\frac{N-1}{2}\sum_{j,k=1}^{n}\sigma^{jk}s_{jk}\prod_{j<k=1}^{n} ds_{jk}$$

In this expression, (a) K is a constant involving only N and n; (b) Σ is the population covariance matrix; (c) S is the sample covariance matrix; (d) the elements of the inverse matrix Σ^{-1} are represented by ∂^{jk} (Harman, 1976). The ∂ in the above equation serves as the likelihood function, L, for the sample. Given this relationship, the next portion of the process involves estimating values for $\hat{A}$ and $\hat{U}^2$ that satisfy the following relationship (Harman, 1976):

$$\Sigma = AA' + U^2$$

The objective is to maximize the value of L. Where A is the matrix of common factor coefficients, and U^2 is a diagonal matrix of "uniqueness" (Harman, 1976, p. 201). This distribution function serves as a basis for a likelihood function (L); this function is given by (Harman, 1976, p. 201):

$$\log L = -\frac{N-1}{2}(\log\left|\sum\right|) + \sum_{j,k=1}^{n}\sigma^{jk}S_{jk} + \text{function independent } \Sigma$$

The maximization of L implies the minimization of the following expression (Harman, 1976):

$$-\frac{2}{N-1}\log L = \log\left|\sum\right| + \sum_{j,k=1}^{n}\sigma^{jk}S_{jk} + \text{function independent } \Sigma$$

The next step in the maximum likelihood estimation process involves finding the partial derivatives with respect to a_{jp} and U_j and equating these expressions to zero; these calculations include "nm+n variables in all." Because the "estimated factor loadings for each variable are proportional the standard deviation of that variable," the estimation equations are scale independent (Harman, 1976, p. 201). The following equations present the results of the estimation procedures in matrix form (Harman, 1976):

$$\hat{p} = \hat{A}\hat{A}' + \hat{U}^2$$

$$\hat{A} = \hat{P}R^{-1}\hat{A}$$

$$\hat{U}^2 = I - diag\hat{A}\hat{A}'$$

$$\hat{A}'R^{-1}\hat{A} = \text{a diagonal matrix}$$

Where P is the population correlation matrix, $\hat{P}$ is an estimator of the population correlation matrix, and R is the sample correlation matrix with ones on the main diagonal (Harman, 1976).

Although the process described above will provide a basis for developing maximum likelihood estimates of factor loadings, Harman (1976) suggests assuming an equivalency between the sample correlation matrix and the estimator of the population correlation matrix. This assumption yields a simpler process for obtaining factor loading estimates. The expression for $\hat{P}$ can be rewritten as (Harman, 1976):

$$AA' + U^2 = R$$

The following expression results from pre-multiplying both sides of the equation by $A'U^{-2}$

$$(A'U^{-2}A + I)A' = A'U^{-2}R$$

We define matrix J as: $J = A'U^{-2}A$

With this definition, the following equation can be formed:

$$(I + J)\,A' = A'U^{-2}R$$

By simplifying the above equation, Harman (1976) describes the following expression as "amenable to an iterative method of solution" $J\,A' = A'U^{-2}R - A'$

The vector of factor loadings is given by: $A = (\mathrm{a}_1\ \mathrm{a}_2 \ldots \mathrm{a}_m)$ where each column vector, m, can be defined as:

$$\mathrm{a}_p = (\mathrm{a}_{1p}\ \mathrm{a}_{2p} \ldots \mathrm{a}_{mp})\ (\mathrm{p} = 1, 2, \ldots \mathrm{m})$$

The iterative process for determining the matrix of factor loadings begins with trial values of a_p. As described by Harman (1976), the products of this process are termed b_p; represents the complete pattern matrix, and V^2 is the resultant matrix of residuals. The following are the iteration equations that yield trial values of a_p:

$$\mathrm{b}_1 = \frac{RU^{-2}a_1 - a_1}{\sqrt{(a_1'U^{-2}(RU^{-2}a_1 - a_1))}},$$

$$\mathrm{b}_2 = \frac{RU^{-2}a_2 - a_2 - b_1b_1'U^{-2}a_2}{\sqrt{(a_2'U^{-2}(RU^{-2}a_2 - a_2 - b_1b_1'U^{-2}a_2))}},$$

$$\mathrm{b}_2 = \frac{RU^{-2}a_3 - a_3 - b_2b_2'U^{-2}a_3 - b_2b_2'U^{-2}a_3}{\sqrt{(a_2'U^{-2}(RU^{-2}a_3 - a_3 - b_1b_1'U^{-2}a_3 - b_2b_2'U^{-2}a_3))}}$$

$$V^2 = I - \text{diag}\,BB'$$

While the above equations illustrate a three factor pattern, the process can be generalized to any number of factors (Harman, 1976). This process is repeated until the algorithm converges on a solution, a matrix that fulfils a prescribed degree of closeness. The resulting matrix, A, contains maximum likelihood estimates of factor loadings (Harman, 1976).

III. ANALYSIS AND DISCUSSION

TABLE 1 LOADING FACTORS WITHOUT OMISSION OF VARIABLES

Variables	Ordinary Least Square Extraction Method					Maximum Likelihood Extraction Method				
	MR1	MR2	h2	u2	com	ML1	ML2	h2	u2	com
GDP	0.72	0.69	1.00	0.00	2.00	0.71	0.70	1.00	0.00	2.00
AGR	0.71	0.69	0.99	0.01	2.00	0.70	0.71	0.99	0.01	2.00
COG	0.68	0.71	0.98	0.02	2.00	0.68	0.71	0.97	0.03	2.00
BC	0.74	0.66	0.99	0.01	2.00	0.73	0.68	0.99	0.01	2.00
WR	0.76	0.65	1.00	0.00	2.00	0.75	0.66	1.00	0.00	2.00
TC	0.89	0.44	0.99	0.01	1.50	0.89	0.45	1.00	0.00	1.50
FI	0.82	0.58	1.00	0.00	1.80	0.80	0.59	1.00	0.00	1.80
EDU	0.69	0.72	0.99	0.01	2.00	0.67	0.74	0.99	0.01	2.00
HLT	0.26	0.51	0.32	0.68	1.50	0.26	0.51	0.32	0.68	1.50
SS Loading	4.62	3.63				4.49	3.75			
Proportion Var	0.51	0.40				0.50	0.42			
Cumulative Var	0.51	0.92				0.50	0.92			
Proportion Explained	0.56	0.44				0.54	0.46			
Cumulative Proportion	0.56	1.00				0.54	1.00			
BIC	123.21					121.1				

TABLE 2 LOADING FACTORS WITH THE OMISSION OF VARIABLE (HLT)

Variables	Ordinary Least Square Extraction Method					Maximum Likelihood Extraction Method				
	MR1	**MR2**	**h2**	**u2**	**com**	**ML1**	**ML2**	**h2**	**u2**	**com**
GDP	0.79	0.62	1.00	0.00	1.90	0.78	0.62	1.00	0.00	1.90
AGR	0.78	0.61	0.99	0.01	1.90	0.79	0.60	0.99	0.01	1.90
COG	0.80	0.58	0.98	0.02	1.80	0.79	0.59	0.97	0.03	1.90
BC	0.76	0.64	0.99	0.01	1.90	0.76	0.64	0.99	0.01	1.90
WR	0.74	0.67	1.00	0.00	2.00	0.75	0.66	1.00	0.00	2.00
TC	0.56	0.83	0.99	0.01	1.80	0.56	0.83	1.00	0.00	1.80
FI	0.68	0.73	1.00	0.00	2.00	0.69	0.72	1.00	0.00	2.00
EDU	0.80	0.59	0.99	0.01	1.80	0.81	0.57	0.99	0.01	1.80
SS Loading	4.43	3.50				4.45	3.47			
Proportion Var	0.55	0.44				0.56	0.43			
Cumulative Var	0.55	0.99				0.56	0.99			
Proportion Explained	0.56	0.44				0.56	0.44			
Cumulative Proportion	0.56	1.00				0.56	1.00			
BIC	117.56					115.08				

From table 1 above, it can be observed that the GDP, AGR, BC, WR, TC and FI have factor loading around 0.72, 0.71, 0.76, 0.89 and 0.82 respectively on factor MR1 while COG, EDU and HLT have factor loading around 0.71, 0.72 and 0.51 respectively on factor MR2 under ordinary least square method of extraction of factor analysis. Also it can be seen that GDP, BC, WR, TC and FI have factor loading around 0.71, 0.73, 0.75, 0.89, and 0.80 respectively on factor ML1 while AGR, COG, EDU and HLT have loading around 0.71, 0.71, 0.74 and 0.51 respectively on factor ML2 under maximum likelihood method of factor analysis. For diagrammatical explanation see figure 1and 2.

Column h2 is the communality of the variables; this is calculated like coefficient of determination R^2 from regression analysis. Variable HLT happen to be the only variable with communality less than 0.5, also variable HLT revealed the highest contribution to the variable GDP from the regression analysis carried out and the regression equation is as follow:

GDP=0.00537+1.3AGR+COG+1.78BC+1.97WR+3.71TC-6.77FI-0.29EDU+9.186HLT *

Consequent upon this, the variable HLT is omitted and the extraction is carry out again on the remaining variables.

Table 2 above revealed that GDP, AGR, COG, BC, WR and EDU have factor loading around 0.79, 0.78, 0.80, 0.76, 0.74, and 0.80 respectively on factor MR1 while TC, and FI have factor loading around 0.83 and 0.73 respectively on MR2 under ordinary least square extraction method of factor analysis. Similarly, GDP, AGR, COG, BC, WR and EDU have factor loading around 0.78, 0.79, 0.79, 0.76, 0.75, and 0.81 respectively on factor ML1 while TC, and FI have factor loading around 0.83 and 0.72 respectively on ML2 under maximum likelihood extraction method of factor analysis. For diagrammatical explanation see figure 3 and 4. The communalities of each variable are greater than 0.5, this affirmed that there is no difference between the original correlations and reproduced correlations and proved that the factor that were extracted accounted for a great deal of variance and the factors do a good job of representing the original data.

Fig.1 OLS extraction
Method without Omission

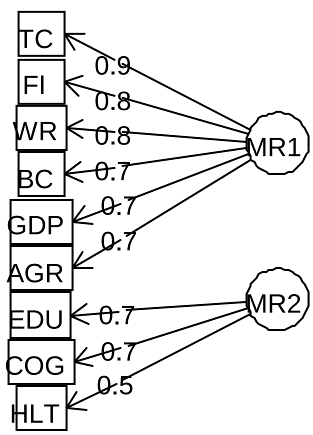


Fig.2 ML extraction
Method without Omission

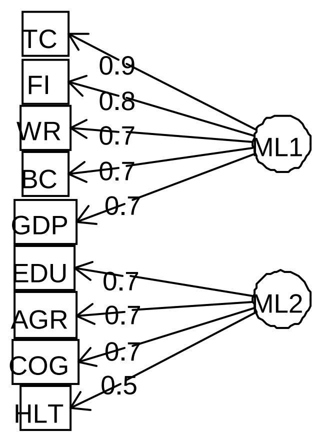


Fig.3 OLS extraction
Method with Omission

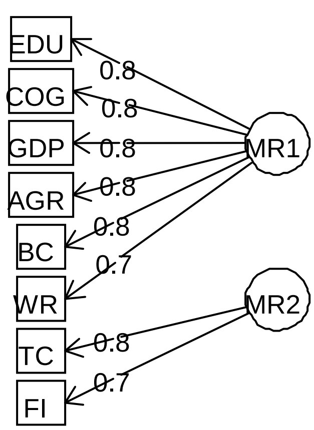
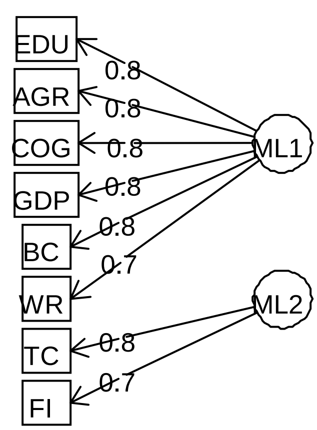


Fig.4 ML extraction
Method with Omission



IV. CONCLUSION

The analysis was conducted under two approaches. Firstly, the sets of data (variables) employed was assumed to be independent of error (i.e endogeneity assumption of ordinary least square is not violated) and the factor loading pattern from both ordinary least square and maximum likelihood method of extraction under this approach are found to different from one and the other (see table 2, figure 1and 2). Examine the communality with each variable, it can be found that communality with variable HLT less than 0.5, also the regression equation (*) revealed that variable HLT contributed most to the GDP. Consequent upon this, variable HLT is omitted to violate the endogeneity assumption. The second analysis from table 3, figure 1 and 2 present the factors loading when the variable HLT which contributed highest unit among the regressors to the GDP in the regression line (*) is omitted (i.e violation of endogeneity assumption). The loading pattern of the both extraction under the violation of endogeneity assumption (second approach) is found to be similar. examine u2 column which is the diagonal element of residuals correlation matrix of the original correlation and reproduced correlation, it can be observed that the factor extracted accounted for a great deal of variance and the factors do a good job of representing the original data, compare to the first approach. Besides, based on Bayesian information criterion, the maximum likelihood method of extraction is found to be lesser under maximum likelihood method of extraction of factor analysis compare to ordinary least square method. Therefore, maximum likelihood outperformed ordinary least square method of extraction of factor analysis.

REFERENCES

[1] Coughlin, Kevin Barry, An Analysis of Factor Extraction Strategies: A Comparison of the Relative Strengths of Principal Axis, Ordinary Least Squares, and Maximum Likelihood in Research Contexts that Include both Categorical and Continuous Variables (2013). Graduate Theses and Dissertations. http://scholarcommons.usf.edu/etd/4459

[2] S.E Finkel, and H.R Ernst, " Civic education in post-apartheid South Africa: Alternative paths to the development of political knowledge and democratic values," *Political Psychology*, Vol. 26, No.3, pp.333-364, 2005.

[3] H.H Harman, Modern factor analysis (1976). (3rd ed.). Chicago: University of Chicago Press.

[4] Spearman,C, "General intelligence, objectively determined and measured," *The American Journal of Psychology*, Vol. 15, No.2, pp. 201-292, 1904.

[5] L.L Thurstone, "The factorial isolation of primary abilities" *Psychometrika*, Vol. 1, pp. 175-182, 1936.

[6] C. Vanzile-Tamsen, L. Harlow and Livingston J, "A measurement model of women's behavioral risk taking," *Health Psychology*, Vol. 25, No.2, pp. 249-254, 2006.

[7] G. Winter ,T. Van Der Westhuizen ,VJ Higgins and C. Curtin M. , Ugliano Contribution of cysteine and glutathione conjugates to the formation of the volatile thiols 3-mercaptohexan-1-ol (3MH) and 3-mercaptohexyl acetate (3MHA) during fermentation by Saccharomyces cerevisiae. Aust J Grape Wine Res 2011, Vol. 17**:** pp. 285–290. 10.1111/j.1755-0238.2011.00127.x

[8] C. Zygmont, and M.R Smith, . "Robust factor analysis in the presence of normality violations, missing data, and outliers: Empirical questions and possible solutions," *The Quantitative Methods for Psychology* ,Vol. 10 , No. 1, pp. 40-55, 2014 .