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Creep Modeling in An Orthotropic FGM Cylinder

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Abstract **- A mathematical model has been developed to estimate steady state creep in an orthotropic cylinder made of functionally graded composite. The FG cylinder is assumed to undergo creep according to Power law. The model developed has been used to investigate the steady state creep response of the FGM cylinder for varying orthotropicity of the material. The results obtained are compared with those estimated for a similar FGM cylinder but having isotropic properties. The result reveals that the presence of orthotropicity in the FGM cylinder may significantly modify its creep response.**

Keywords : **Modeling, Steady state creep, Cylinder, Functionally Graded Material, Orthotropic**

I. Introduction

Functionally graded materials (FGMs) are microscopically inhomogeneous composite materials in which the volume fraction of two or more materials is varied smoothly and continuously as a function of position in certain direction(s) of the structure from one point to another [6, 13]. These materials are mainly constructed to operate in high temperature environments and are made from a mixture of metal and ceramic or a combination of different metals. FGMs have been developed as ultra high temperature resistant materials for potential applications in aircrafts, space vehicles and other structural components exposed to elevated temperature [5].

Bhatnagar *et al.* [2] have presented the analysis for an orthotropic thick-walled cylinder undergoing creep due to the combined action of internal and external pressures, and rotary inertia. It is observed that the cylinder with more strength in the radial direction has lower effective stress and performs better. Gupta *et al.* [4] analyzed creep stresses and strain rates in a rotating non-homogeneous thick-walled cylinder by using Seth's transition theory. The study indicates that for a rotating non-homogeneous cylinder, with compressibility increasing radially, the circumferential stress is maximum at the external surface at lesser angular speed but at higher angular speed it becomes maximum at the internal surface. The compressive value of axial stress, observed at the external surface, increases with the increase in angular speed. Chen *et al.* [3] analyzed creep behavior of an FGM cylinder subjected to both internal and external pressures. It was assumed that the properties of graded material are axi-symmetric and depend on the radial coordinate. An asymptotic solution was derived on the basis of Taylor expansion series. The approximate solutions calculated by taking different higher-order terms in the Taylor series were compared with the results of finite element (FE) analysis performed in ABAQUS software. It is observed that although the use of higher-order terms may help to obtain a more accurate result for the time-dependent behavior of the cylinder, a fifth-order form is sufficiently accurate to calculate the distribution of creep stress with satisfactory approximation. Singh and Gupta [9] investigated the steady state creep in a transversely isotropic functionally graded cylinder operating under internal and external pressures. They described creep behavior of the cylinder by a threshold stress based creep law. The effect of anisotropy was investigated on the creep stresses and creep rates in the FGM cylinder. The study reveals that in the presence of anisotropy, the radial and tangential stresses are marginally affected whereas the axial and effective stresses vary significantly. The strain rates as well as strain rate inhomogeneity decrease significantly when the extent of anisotropy (α) reduces from 1.3 to 0.7. Sadeghi *et al* [7] carried out strain gradient elasticity formulation to analyze FG micro-cylinders. The material properties were assumed to obey a power law distribution in the radial direction. A power series solution for stresses and displacements in FG micro-cylinders subjected to internal and external pressures was obtained. Numerical examples were presented to study the effect of characteristic length parameter and FG power index on the displacement field and stress distribution in the FG cylinders. It is observed that the characteristic length parameter has a considerable effect on the stress distribution of FG micro-cylinders. The increase of material length parameter leads to decrease the maximum radial and tangential stresses in the cylinder. The study also reveals that the FG power index has a significant effect on the maximum radial and tangential stresses.

The literature consulted so for reveals that a number of studies have been undertaken to investigate the creep behavior of composite cylinders. The studies pertaining to creep behavior of FGM cylinder are rather limited. Further, the studies on FGM cylinders, however limited, assume the material to be isotropic. In actual, the FGMs are anisotropic in nature. The anisotropy may be induced during processing such as forging or due to initial creep deformation. Therefore, it is imperative to consider the effect of anisotropy on the creep behavior of the FGM cylinder subjected to thermo-mechanical loading.

II. Distribution of Reinforcement

The cylinder is made of Al-SiC_p composite with SiCp content decreasing linearly from the inner to outer radius. Therefore, the density and the value of creep parameters *B* and *n* will vary with the radial distance. The content (vol. %)

of SiC_p, V(*r*), at any radius *r* of the cylinder is given by Singh and Gupta [9],

$$V(r) = V_{max} - \frac{(r-a)}{(b-a)}\left[V_{max} - V_{min}\right] \quad (1)$$

Where V_{max} and V_{min} are respectively the maxim um (at the inner radius) and t he minimum (at the outer radius) SiCp content in the cylinder.

The average SiCp content in the cylinder can be expressed as,

$$V_{avg} = \frac{\int_a^b 2\pi r l V(r) dr}{\pi(b^2 - a^2)l} = \frac{2\int_a^b r V(r) dr}{(b^2 - a^2)} \quad (2)$$

Where *l* is the length of cylinder.

Substituting V(r) from Eq. (1) into Eq. (2) and integrating, we get,

$$V_{min} = \frac{3V_{avg}\left(1-\lambda^2\right)\left(1-\lambda\right) - V_{max}\left(1-3\lambda^2+2\lambda^3\right)}{2-3\lambda+\lambda^3} \quad (3)$$

Where $\lambda = (a/b)$ is the ratio of inner to outer radius of the cylinder.

III. Creep Law And Parameters

The creep behavior of the FGM cylinder is described by Norton's power law.

$$\dot{\varepsilon}_e = B\sigma_e^n \quad (4)$$

Where $\dot{\varepsilon}_e$ is the effective strain rate, σ_e is the effective stress, *B* and *n* are material parameters describing the creep performance in the cylinder.

It is evident from the study of Singh and Ray [8] that the values of creep parameters *B* and *n* appearing in Norton's law depend on the content of reinforcement, which vary with the radial distance. It is also revealed that the effect of varying SiC_p content on the creep parameters *B* and *n* is opposite to each other. The value of *B* decreases with the decrease in SiC_p content but the value of *n* increases with the decrease in SiC_p content. In the light of this, the values of Power law multiplier (*B*) and stress exponent (*n*) appearing in the creep law (Eq. 4), at any radius r of the FGM cylinder are estimated by following equations.

$$B(r) = B_o\left[V(r)/V_{avg}\right]^{\phi} \quad (5)$$

and

$$n(r) = n_o\left[V(r)/V_{avg}\right]^{-\phi} \quad (6)$$

Where B_o and no are respectively the values of creep parameters *B* and *n* respectively and ϕ is the grading index. The values of B_o, n and ϕ are the taken from the study of Chen *et al.* [3].

TABLE I VALUES OF CREEP PARAMETERS [3] AND DIMENSION OF THE MODEL

Creep parameters	Dimension of the Model
B_o=2.77 x 10^{-16} (MP^{-n}/h) n = 3.75	Inner radius (*a*) =10 *mm* Outer radius (*b*)= 20*mm*
Grading index ϕ = 0.7	Thickness (*t*) = 1 *mm* Length (*l*) = 20*mm*

IV. Mathematical Formulation

Let us consider a thick-walled hollow cylinder made of functionally graded Al-SiC_p composite having inner and outer radii as *a* and *b* respectively. The cylinder is subjected to internal and external pressures denoted respectively by *p* and *q*.

For the purpose of analysis the following assumptions are made:

(i)The material of the cylinder is orthotropic and incompressible i.e. $\dot{\varepsilon}_r + \dot{\varepsilon}_\theta + \dot{\varepsilon}_z = 0$

where r, θ and z are taken respectively along the radial, tangential and axial directions of the cylinder.

(ii) The cylinder is subjected to internal pressure that is applied gradually and held constant during the loading history.

(iii) Elastic deformations in the cylinder are neglected as compared to creep deformations.

The cylinder is sufficiently long and hence is assumed under plain strain condition (i.e. axial strain rate, $\dot{\varepsilon}_z = 0$)

The radial ($\dot{\varepsilon}_r$) and tangential ($\dot{\varepsilon}_\theta$) strain rates in the cylinder are given by:

$$\dot{\varepsilon}_r = \frac{d\dot{u}_r}{dr} \quad (7) \qquad \text{and} \qquad \dot{\varepsilon}_\theta = \frac{\dot{u}_r}{r} \quad (8)$$

Where $\dot{u}_r = (du/dt)$ is the radial displacement rate and u is the radial displacement.

Eqs (7) and (8) may be solved to get the following compatibility equation,

$$r\frac{d\dot{\varepsilon}_\theta}{dr} = \dot{\varepsilon}_r - \dot{\varepsilon}_\theta \quad (9)$$

The cylinder is subjeconditions,

$$\sigma_r = -p \text{ at } r = a \quad (10)$$

$$\sigma_r = -q \text{ at } r = b \quad (11)$$

Where the negative sign of σ_r implies the compressive nature of radial stress.

By considering the equilibrium of forces acting on an element of the cylinder in the radial direction, we get,

$$r\frac{d\sigma_r}{dr} = \sigma_\theta - \sigma_r \quad (12)$$

Since the material of the cylinder is incompressible, therefore,

$$\dot{\varepsilon}_r + \dot{\varepsilon}_\theta + \dot{\varepsilon}_z = 0 \quad (13)$$

The constitutive equations under multi axial creep in an orthotropic cylinder, when the principal axes are the axes of reference, Bhatnagar and Gupta [1] are given by,

$$\dot{\varepsilon}_r = \frac{\dot{\varepsilon}_e}{(G+H)\sigma_e}\left[G(\sigma_r - \sigma_z) + H(\sigma_r - \sigma_\theta)\right] \quad (14)$$

$$\dot{\varepsilon}_\theta = \frac{\dot{\varepsilon}_e}{(G+H)\sigma_e}\left[F(\sigma_\theta - \sigma_z) + H(\sigma_\theta - \sigma_r)\right] \quad (15)$$

$$\dot{\varepsilon}_z = \frac{\dot{\varepsilon}_e}{(G+H)\sigma_e}\left[F(\sigma_z - \sigma_\theta) + G(\sigma_z - \sigma_r)\right] \quad (16)$$

Where F, G and H are the anisotropic constants, $\dot{\varepsilon}_e$ and σ_e are respectively the effective strain rate and effective stress in the FGM cylinder.

The Hill's yield criterion, when the Principal axes of anisotropy are the axes of reference, Dieter [11], is given by,

$$\sigma_e = \left[\frac{1}{(G+H)}\left\{F(\sigma_\theta - \sigma_z)^2 + G(\sigma_z - \sigma_r)^2 + H(\sigma_r - \sigma_\theta)^2\right\}\right]^{1/2} \quad (17)$$

Under plain strain condition ($\dot{\varepsilon}_z = 0$), one may get from Eqs. (7), (8) and (13),

$$\dot{u}_r = \frac{C}{r} \quad (18)$$

Where C is a constant of integration. Using Eq. (18) in Eqs. (7) and (8), we get,

$$\dot{\varepsilon}_r = -\frac{C}{r^2} \quad (19) \quad \text{and} \quad \dot{\varepsilon}_\theta = \frac{C}{r^2} \quad (20)$$

Under plane strain condition, Eq. (16) becomes,

$$\sigma_z = \frac{(G\sigma_r + F\sigma_\theta)}{(F+G)} \quad (21)$$

Substituting σ_z from Eq. (21) in to Eq. (17), we get,

$$\sigma_e = \frac{(\sigma_\theta - \sigma_r)}{\sqrt{(H+G)}}\left[\frac{(FG+GH+HF)}{(F+G)}\right]^{1/2} \quad (22)$$

Substituting $\dot{\varepsilon}_r$ and σ_z respectively from Eqs. (19) and (21) into Eq. (14), we obtain,

$$\sigma_\theta - \sigma_r = \frac{(F+G)(H+G)}{(FG+GH+HF)}\frac{\sigma_e}{\dot{\varepsilon}_e}\frac{C}{r^2} \quad (23)$$

Using Eqs. (4) and (22) in Eq. (23) and simplifying, one gets,

$$\sigma_\theta - \sigma_r = \frac{I_1}{r^{2/n}} \quad (24)$$

Where, $$I_1 = \left[\frac{(F+G)(H+G)}{(FG+GH+HF)}\right]^{\frac{n+1}{2n}}\frac{C^{1/n}}{B^{1/n}} \quad (25)$$

Substituting Eq. (24) into Eq. (12) and integrating, we get,

$$\sigma_r = X_1 - p \quad (26)$$

Where, $$X_1 = \int_a^r \frac{I_1}{r^{\frac{n+2}{n}}}dr \quad (27)$$

Substituting Eq. (26) into Eq. (24), we obtain,

$$\sigma_\theta = X_1 + \frac{I_1}{r^{2/n}} - p \quad (28)$$

To estimate the value of constant C, needed for estimating , the boundary conditions given in Eqs. (10) and (11) are used in Eq. (26) with X_1(Eq. 27) integrated between limits a to b. to get,

$$\int_a^b \frac{I_1}{r^{\frac{n+2}{n}}}d - p = -q \quad (29)$$

Substituting the value of I_1 from Eq. (25) in to Eq. (29) and simplifying, we obtain,

$$C = \left[\frac{p-q}{X_2}\right]^n \quad (30)$$

Where, $$X_2 = \int_a^b \frac{(T)^{\frac{n+1}{n}}}{r^{\frac{n+2}{n}}B^{\frac{1}{n}}}dr \quad \text{and} \quad T = \sqrt{\frac{(F+G)(H+G)}{(FG+GH+HF)}} \quad (31)$$

Using Eqs. (21) and (22) into Eqs. (14) and (15), one obtains,

$$\dot{\varepsilon}_\theta = -\dot{\varepsilon}_r = \frac{\dot{\varepsilon}_e}{\sqrt{\frac{(F+G)(H+G)}{(FG+GH+HF)}}} \quad (32)$$

The analysis presented above yields the results for isotropic FGM cylinder. When the anisotropic constants are set equal i.e. F=G=H.

V. Estimation Of Anisotropic Constants

The Hill's yield criterion for orthotropic material, as given by Eq. (17), involves constants F, G and H, the values of which are required for estimating involve creep response of the FGM cylinder. If X, Y and Z are the tensile stresses in the principal directions of anisotropy, then according to Hill, Dieter[11].

$$\left[\frac{1}{X^2} = G+H; \frac{1}{Y^2} = H+F; \frac{1}{Z^2} = F+G\right] \quad (33)$$

The above set of equations may be solved to estimate the values of anisotropic constants as given below,

$$\begin{bmatrix} 2F = \left(\frac{1}{Y^2}+\frac{1}{Z^2}-\frac{1}{X^2}\right); 2G = \left(\frac{1}{Z^2}+\frac{1}{X^2}-\frac{1}{Y^2}\right); \\ 2H = \left(\frac{1}{X^2}+\frac{1}{Y^2}-\frac{1}{Z^2}\right) \end{bmatrix} \quad (34)$$

For isotropic material the ratio of anisotropic constants is unity i.e. F/G = G/H = H/F = 1.

If the material of cylinder is subjected to uniaxial loading in r and θ directions, the corresponding stress invariant may be expressed in terms of observed tensile strength and Hill's anisotropic constants as given below,

$$\sigma_e = \sqrt{\frac{G+H}{2}}\sigma_{r_y} \quad (35) \text{ and } \quad \sigma_e = \sqrt{\frac{F+H}{2}}\sigma_{\theta_y} \quad (36)$$

where $\sigma_{r_Y}, \sigma_{\theta_Y}$ are respectively the yield strength of composite in r and θ directions and σe is the isotropic yield stress. If the material of cylinder is tested under uniaxial loading in z direction, the stress invariant may similarly be written as,

$$\sigma_e = \sqrt{\frac{F+G}{2}}\sigma_{z_y} \quad (37)$$

where σ_{z_y} is the yield strength of composite in z direction.

It is assumed that during processing of FGM cylinder the whiskers get aligned in the tangential (θ) direction, leading to anisotropic behavior. Therefore, in FGM cylinder the direction θ becomes longitudinal direction and the remaining directions (i.e. r and z) may be taken as transverse directions. For axisymmetric problems like cylinder, the directions r, θ and z may be taken as the principal directions. Thus, the anisotropic constants given by Eqn. (34), may be expressed as,

$$F = \left(\frac{1}{\sigma_{\theta_y}{}^2}+\frac{1}{\sigma_{z_y}{}^2}-\frac{1}{\sigma_{r_y}{}^2}\right)\sigma_e{}^2 \quad (38)$$

$$G = \left(\frac{1}{\sigma_{z_y}{}^2}+\frac{1}{\sigma_{r_y}{}^2}-\frac{1}{\sigma_{\theta_y}{}^2}\right)\sigma_e{}^2 \quad (39)$$

$$H = \left(\frac{1}{\sigma_{r_y}{}^2}+\frac{1}{\sigma_{\theta_y}{}^2}-\frac{1}{\sigma_{z_y}{}^2}\right)\sigma_e{}^2 \quad (40)$$

When assume the $\alpha = \sigma_{r_y}/\sigma_{\theta_y}$ and $\beta = \sigma_{z_y}/\sigma_{\theta_y}$ The value of G/F > 1 and H/F < 1 implies that the yield strength of FGM cylinder in the tangential direction is the highest and lowest in the axial direction. On the contrary, G/F < 1 and H/F > 1 imply that the yield strength of the cylinder is the highest in tangential direction but lowest in the tangential direction.

To study the effect of anisotropy on the stress and strain rates, following numerical values of anisotropic constants taken from Kulkarni *et al.* [10] has been used.

Table II Values Of Anisotropic Constants Taken From [10]

	FGM Cylinder C1(case 1)	FGM Cylinder C2 (case 2)	FGM Cylinder C3 (case 3)
G/F	1.22	1	0.7452
H/F	.7452	1	1.22

VI. Numerical Scheme Of Computation

To begin the computation process, the vlaue of X_2 is estimated from Eq.(31) after substituting the value of anisotropic constants F,G and H and the values of creep parameters B and n, as estimated from Eqs. (5) and (6) respectively.Thereafter,the value of constant C is estimated from Eq. (30) and using this in Eq.(25) the value of I1 is obtained. The value of I_1, thus estimate is used in eq. (27) to get X_1.Knowing X_1, the stresses σ_r and σ_θ are obtained respectively from Eqs. (26) and (28).The values of and are substituted in Eq. (21) to estimate the distribution of axial stress (σ_z) in the cylinder. Knowing σ_r, σ_θ and σ_z, the values of and are obtained respectively from Eqs. (4) and (22). Finally, the strain rates $\dot{\varepsilon}_r$ and $\dot{\varepsilon}_\theta$ are estimated respectively from Eqs. (14) and (15).

VII. Results And Discussion

A. Validation

Before discussing the results obtained in this study, it is considered necessary to validate the analytical procedure used in this study. The values of *B* and *n* are assumed to be constant for the cylinder as 2.77×10-[16] MPa-[n]/h and 3.75, similar to the work of Chen et al.[3].The distribution of tangential stress estimated in the cylinder is compared with those reported by Chen *et al.*[3], in Fig. 1. An excellent agreement is observed between these results, validating the present study.

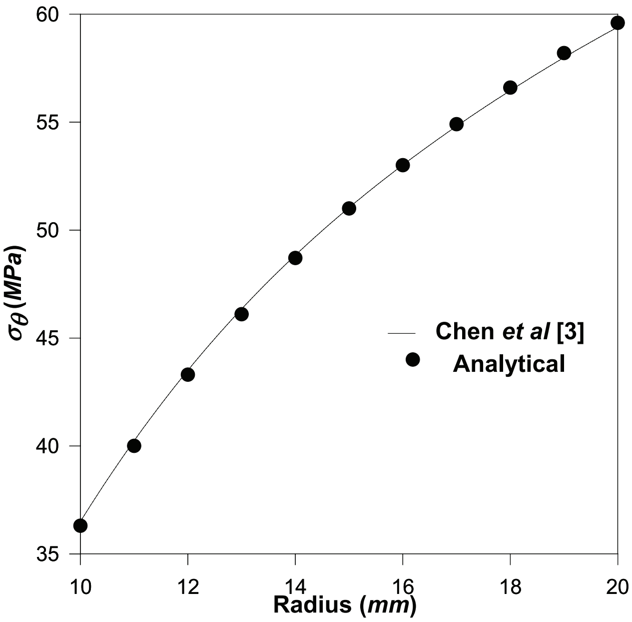

Fig. 1 Validation of Chen *et al.* [3] vs. Analytical

B. Variation of Creep Parameters

Figure 2 shows the variation of creep parameters *B* and *n* with radial distance in FGM cylinders. The value of parameter *B* and *n* in the FGM cylinder is supposed to decrease and increase respectively with increase in radial distance, as is evident from the Eqs. (5) and (6). The variations observed in parameters *B* and *n* are attributed to decreasing SiC_p content, V(*r*), in the FGM cylinders with increasing radius (r), as evident from Eq. (1). Owing to similar distribution of reinforcement (SiC_p) in the different FGM cylinders C1-C3, they have similar variations of parameters *B* and *n*.

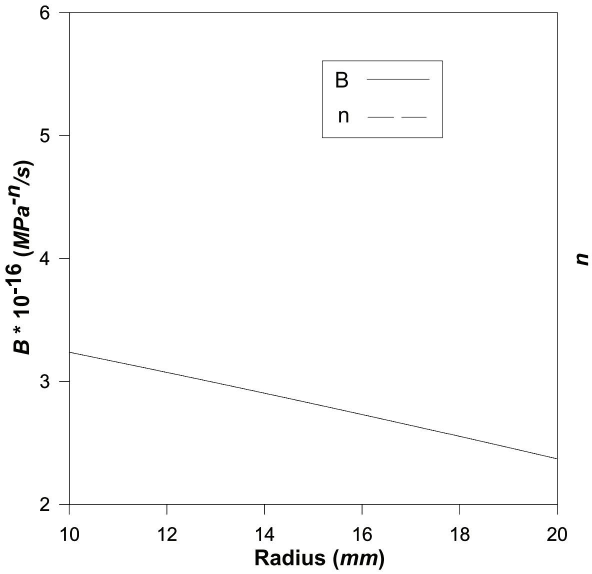

Fig. 2: Variation of creep parameters B and n in cylinder

C. Effect of Anisotropy on Stresses and Strain Rates

Figure 3 shows the effect of anisotropy on radial stress in the FGM cylinders. The radial stress remains compressive over the entire cylinder with a maximum (compressive) and zero values reported at the inner and outer radii respectively, under the imposed boundary conditions given in Eqs. (10) and (11). the results obtained through analytical technique are not affected by varying degree of anisotropy in the FGM cylinders. This is attributed to the fact that the term X*1* used in Eq. (26) for calculating radial stress is not affected by varying the extent of anisotropy in the FGM cylinders.

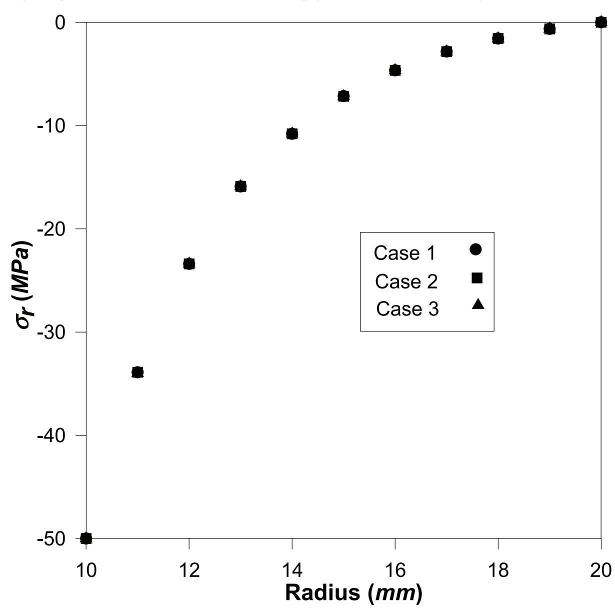

Fig.3 Effect of anisotropy on radial Stress

Figure 4 shows the effect of anisotropy on tangential stress in the FGM cylinder. The tangential stress remains tensile throughout the FGM cylinders and is observed to decrease with the increase in radius. The results obtained by analytical procedure reveals that the tangential stress in the FGM cylinder is not affected by varying the extent of anisotropy in the FGM cylinders. This is attributed to the fact that the terms X_1 and $I_1/r2/^n$, used in Eq. (28) for calculating tangential stress, are not dependent on the extent of anisotropy.

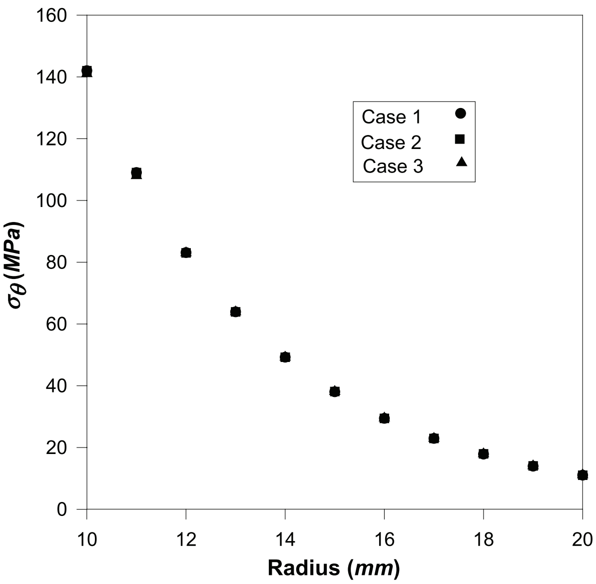

Fig. 4 Effect of anisotropy on Tangential Stress

Figure 5 shows the effect of anisotropy on effective stress in the FGM cylinder. The effective stress decreases with increasing radial distance. The results of analytical procedure reveal that the effective stress is observed to be minimum for FGM cylinder C1 and maximum for FGM cylinder C3 when compared with the isotropic FGM cylinder C2.

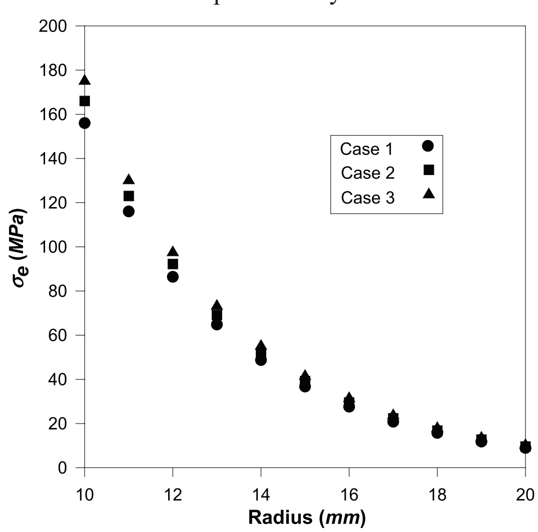

Fig. 5 Effect of anisotropy on Effective Stress

Figure 6 shows the effect of anisotropy on radial and tangential strain rates in FGM cylinders. The radial and tangential strain rates in the cylinder are equal in magnitude but opposite in nature under the assumptions of incompressibility (Eq. 13) and plain strain condition ($\dot{\varepsilon}_z = 0$). The effect of anisotropy on radial and tangential strain rates in the FGM cylinder decreases with increasing radius. The radial strain rate is the lowest in FGM cylinder C1 and the highest in FGM cylinder C3 when compared to isotropic FGM cylinder C2. The effect of anisotropy on effective strain rate (Fig.7) is observed to be similar as for tangential strain rate in Fig.6.

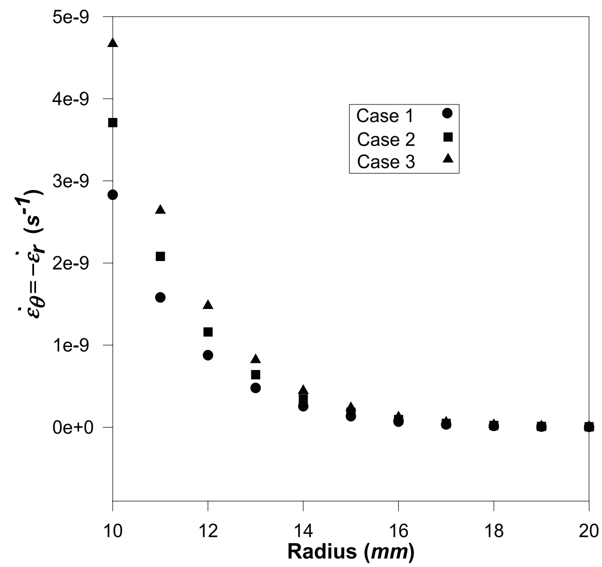

Fig. 6 Effect of anisotropy on radial and tangential strain rate

The effect of anisotropy on effective strain rate in the cylinder similar those described for radial and tangential strain rates (refer Fig. 6).

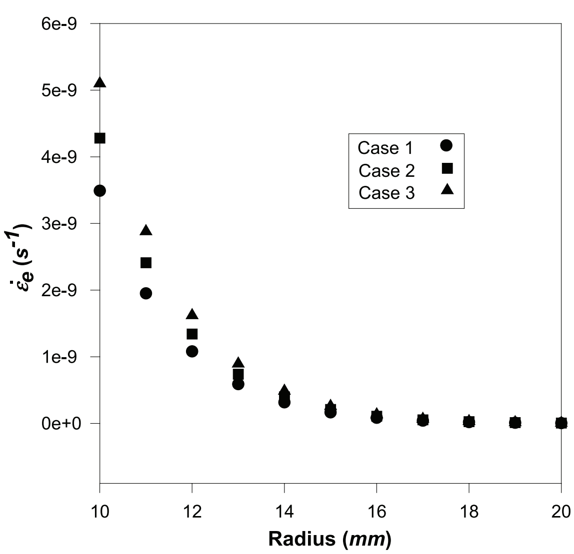

Fig. 7 Effect of anisotropy on effective strain rate

VIII. Conclusions

The present study has led to the following conclusions:

1. The effect of anisotropy on radial and tangential stresses increases near the inner radius but exhibit are decrease towards the outer radius.
2. The effective stresses in the FGM cylinder with $\sigma_{\theta_y} > \sigma_{r_y} > \sigma_{z_y}$ is lower everywhere as compared to any other FGM cylinder. The effect of anisotropy on the effective stress decreases with increasing radius.
3. The strain rates (radial, tangential and effective) in the FGM cylinder is the lowest for the FGM cylinder with $\sigma_{\theta_y} > \sigma_{r_y} > \sigma_{z_y}$ and the highest for the FGM cylinder with $\sigma_{z_y} > \sigma_{r_y} > \sigma_{\theta_y}$.
4. The effect of anisotropy on strain rates decreases with increasing radius.

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