Asian Review of Mechanical Engineering
ISSN: 2249-6289 (P) Vol.3 No.1, 2014, pp.18-29
© The Research Publication, www.trp.org.in
DOI: https://doi.org/10.51983/arme-2014.3.1.2356

Non-Linear Signal Analysis Applied to Surface Wear Condition Monitoring in Reciprocating Sliding Testing Machines

Mohammad Amin Rashidifar[1] and **Ali Amin Rashidifar**[2]
[1]*Faculty of Mechanical Engineering, Islamic Azad University, Shadegan Branch, Shadegan, Iran* [2]*Computer Science, Islamic Azad University, Shadegan Branch, Shadegan, Iran*
E-mail: rashidifar_58@yahoo.com

(Received on 02 March 2013 and accepted on 15 June 2014)

Abstract – **When the surfaces of two elastic bodies present relative motions under certain amount of contact pressure the mechanical system can be instable. Experiments conducted on elastic bodies in contact shown that the dynamical system is self-excited by the non-linear behavior of the frictional forces. The main objective of this paper is to estimate the friction force using the vibrations signals, measured on a reciprocating wear testing machine, by the proposed non-linear signal analysis formulation. In the proposed formulation the global output is the sum of two outputs produced by a linear path associated in parallel with a non-linear path. This last path has a non-linear model that represents the friction force. Since the linear path is identified by traditional signal analysis the non-linear function can be evaluated by the global input/output relationships. Validation tests are conducted in a tribological system composed by a sphere in contact with and a prismatic body, which has an imposed harmonic motion. The global output force is simultaneously measured by a piezoelectric load cells. The sphere and prismatic body vibrations are measured by a laser Doppler vibrometer and by an accelerometer respectively. All signals are digitalized with the same time base and the data is transferred to a microcomputer. The non-linear signal analysis technique uses this data to identify the friction force.**

Keywords: **Non-linear signal analysis, Friction force identification, Wear, Vibration.**

I. INTRODUCTION

Nowadays the analysis of nonlinear dynamical systems and the analysis of nonlinear forces applied to linear systems are increasing of importance. These dynamical systems with nonlinear properties or time dependent properties cannot be analyzed using the theories developed to linear systems. Analysis of these kinds of systems demand complex methodologies to predict its response or even to estimate its dynamical characteristics and models parameters. Even the study of the linear systems could require a nonlinear methodology, are examples linear instrumentation used to measure nonlinear physical quantities or to prediction of the response of the linear system to a nonlinear force as a vibratory system subjected to friction force as excitation force.

Classical signal analysis, used in the study of dynamical system, was developed to stationary random data. However, much of the random data of interest in engineering analysis is nonstationary when viewed as a whole. Typical examples of nonstationary data are vibrations signals measured in dynamical systems subjected to nonlinear forces, in dynamical systems with temperature dependent physical parameters, signals obtained from dynamical systems with cubic stiffness or time varying mass and so. Normally to analyze these kinds of signals the engineers use VOLTERRA series to study nonlinear properties and the time dependent properties. [1]

However, VOLTERRA series provide results that are difficult to be physically interpreted and are difficult to be calculated. Nevertheless these difficulties should be cited that estimated system and the results are very dependent of the probability density functions (PDF) of the input signals in the dynamical systems. Therefore, the results and the systems parameters obtained with inputs that don't have the same PDF nature cannot be compared.

Recently was developed a new methodology based on multiple input/single output (MISO) linear analysis of reverse dynamic systems and on the statistical relations between the input excitation data and the output response data of the dynamical system. This methodology can estimate the nonlinear properties of the dynamical system when the input data and the output data were measured, also estimate the input from the previous knowledge of the systems properties and of the output data or estimate the output if the input was measured. The main advantages of this inverse MISO methodology of nonlinear analysis against the use of VOLTERRA series are the low computational cost and the results that are easy to be physically interpreted. Should be cited that the results are not dependent of the PDF nature of input data since this statistical property is considered on the analysis. [2]

A directly application of the nonlinear MISO analysis is its usage in the measurement of nonlinear forces by means of the linear devices. Since the linear parameters of the instrumentation devices are well know, by means of traditional signal analysis and system identifications methodologies, the nonlinear force can be measured indirectly using the nonlinear MISO analysis applied to the output data. Using this methodology the dynamical system model and its parameters can be analyzed at the same time. Should be cited that to nonlinear models, that are single-valued function, can be identified using the relations between the input and the output data probability distribution functions. [2]

Many engineering problems deal with the measurement of nonlinear forces as examples:

- Tribology: science area interested in measure the friction force.
- Aeronautics: a science that is interested in measure the aerodynamical forces acting on the wings.
- Control: could be interested in friction forces, in magnetic forces and other kind of nonlinear forces.

This paper proposes a methodology to estimate the friction force, acting as excitation of a linear dynamical system or as a Coulomb damper attached to the dynamical system, using the system properties and its output response. This methodology is based on nonlinear MISO analysis, cited before, and on Coulomb law of friction.

Coulomb law of friction was chooses due its wide range of validity when is considered a wide range of tribological conditions. This law of friction has parameters that are easy to be physically interpreted. Additionally should be noted that the nature of friction force do not change even when the values of the kinetic and static friction force changes. [4]

To show these properties of adopted law of friction an experiment was carried in three different tribological conditions. To simulate a dynamical system subjected to a nonlinear friction force an experimental device was build with a rigid plate supported by an elastic suspension, to this device was attached a Coulomb damper device lubricated with mineral oil.

Another group of experiment was conducted on a reciprocating tribometer to study the efficacy of estimated friction force, using the proposed methodology, in the study of solid lubrication performance using a thin Teflon™ layer. So the proposed methodology to estimate friction force, that has its validity tested in the first group of the experiments, is applied to tribological tests to study its sensibility to small changes in the tribological conditions. This study is very useful to tribology, since the researchers of this area believe that changes in surface of contact bodies produces specific changes in the friction force. Therefore the correct estimative of friction force without influence of dynamical behavior of the tribometer is a important application of nonlinear signal analysis. [3]

II. Fundamentals of Non-Linear Signal Analysis

Physical parameters of the nonlinear system cannot be identified by traditional linear analysis since this technique implies that the probability density function of the inputs and outputs should have the same nature, condition not verified in nonlinear systems. [1]

VOLTERRA series theory can be used to represent nonlinear systems using sets of high order polynomial functions assembled in parallel. However, due the large calculation effort and dependencies of the VOLTERRA series to the probability density function (PDF) of input data, Bendat [2] proposed a new methodology that uses the well-known theory of multiple input/multiple output linear systems (MISO) modified to be applied to signal analysis of non-linear systems.

The nonlinear signal analysis is based on the hypothesis that the nonlinear characteristics of the system is additive to its linear characteristics and that the nonlinear effects acting on one input produces instantaneous output.

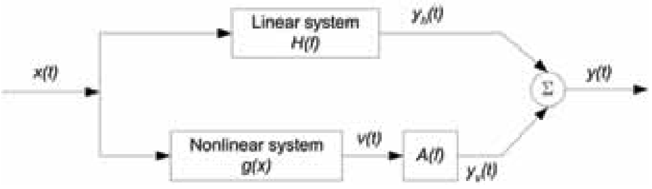

Fig. 1 Nonlinear MISO systems block diagram.

The nonlinear system represented in the Fig.1 does not have limitations regarding the characteristics of the signal input $x(t)$ PDF, as imposed by VOLTERRA models. However to use this methodology a prior knowledge of the nonlinear functions $g(x)$ is required. However, Bendat [2] proposed that a prior analysis of the relation between cumulative probability function of the input data and the output data should estimate the nonlinear function $g(x)$.

As in the linear MISO theory on the non-linear MISO theory statistical uncorrelated inputs produces statistical uncorrelated outputs. This properties can be used to separate the nonlinear and the linear parts from the total output *y(t)*. This implies that the coherence function between the nonlinear input $u(t)$ and the linear input $x(t)$ in the Fig. 2 is null at all frequencies. So the outputs data $y_o(t)$ and $y_u(t)$ are completely uncorrelated and are data produced only by the linear input and only by the nonlinear input respectively.

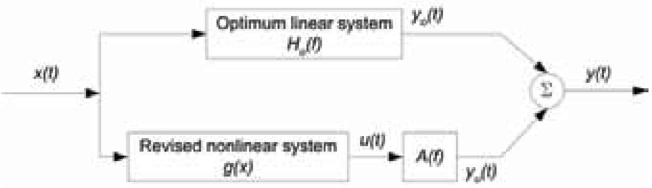

Fig. 2 Nonlinear MISO system with uncorrelated inputs.

After the calculations of the $v(t)=g(x(t))$ the power spectral densities and the coherence functions are calculated by means of:

$$
\begin{array}{lll}
L_{xv}=\dfrac{S_{xv}}{S_{xx}} & S_{uy}=S_{vy}-L_{xv}*S_{xy} & Ho={S_{xy}}/{S_{xx}} \\
S_{y_o y_o}=\left|H_o\right|^2*S_{xx} & S_{uu}=S_{vv}*\left(1-\gamma_{xv}\right) & A={S_{uy}}/{S_{uu}} \\
H=H_o-\left({S_{xy}}/{S_{xx}}\right)*A & S_{\mathrm{y_h y_h}}=\left|H\right|^2*S_{xx} & S_{y_u y_u}=\left|A\right|^2*S_{uu} \\
S_{y_v y_v}=\left|A\right|^2*S_{vv} & \gamma_{xv}={\left|S_{xy}\right|^2}/{S_{xx}*S_{vv}} &
\end{array}
\tag{1}
$$

The system presented by Fig.1 and Fig.2 have only one nonlinear function, however it should be noted that this methodology can represent systems with more than one nonlinear behavior. In this case the nonlinear MISO system representing each nonlinear function is preceded by a linear frequency response function (FRF), which represents the frequency dependency of nonlinear path. The PSD function and the coherence functions presented in Eq. (1) should be extended to more than two paths in parallel. So the use of nonlinear MISO formulation permits an easy correlation of estimated parameters with physical properties of the assumed model to nonlinear systems even in models with more than one nonlinear property. [3]

III. VIBRATORY SYSTEMS WITH NONLINEAR PROPERTIES

To apply the proposed methodology in the analysis of nonlinear dynamical systems should be done a prior mathematical and physical modeling to identify the linear and the nonlinear properties of this system. In the mechanical system shown in Fig.3 the linear parameters m, c and k are mathematical representation of the physical properties mass, viscous damper and spring, respectively. A generic nonlinear device that produces a force $g(x)$ is connected in parallel to the suspension, and an external force $F(t)$ excites the system. The time domain mathematical model to this system is presented in Eq.2 and the correspondent frequency domain model of Eq.3 is obtained by taking the Fourier Transform of the Eq.2.

$$m\ddot{x}+c\dot{x}+kx+ag(x)=F(t) \tag{2}$$

$$F(f)=[H]X(f)+A(f)F(g(x)) \tag{3}$$

The term $A(f)$ represents the frequency dependencies of the nonlinear device and $F(g(x))$ is the Fourier transform of the nonlinear input $v(t)$. Eq. and Eq. show that there are no restriction on the nature of nonlinear function $g(x)$ and that the nonlinear and linear terms are additive, according MISO theory. [2]

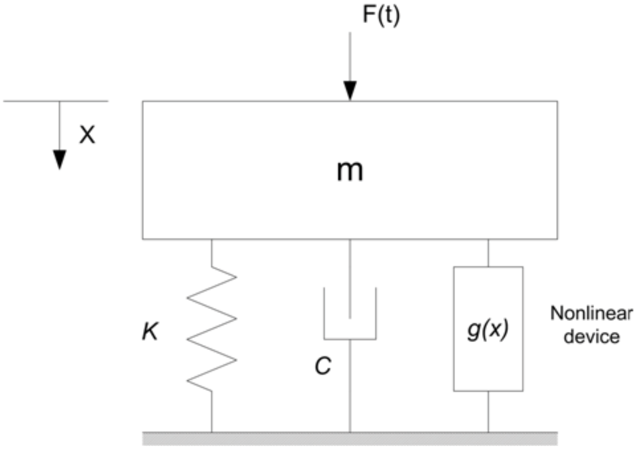

Fig. 3 One degree of freedom mechanical system with a nonlinear device.

The same procedure can be applied to systems with several degrees of freedom and several nonlinear effects without loss of generality. In this paper, the formulation is applied to identify the nonlinear force acting on a linear system, using the measured system response. The analysis of linear systems submitted to friction forces are of great interest to tribology field of research, since in the wear experiments the friction force between the specimens carries information about the surface condition. [3]

A generic linear system excited by a nonlinear force is represented in Fig.4 and its mathematical model is given by Eq.4.

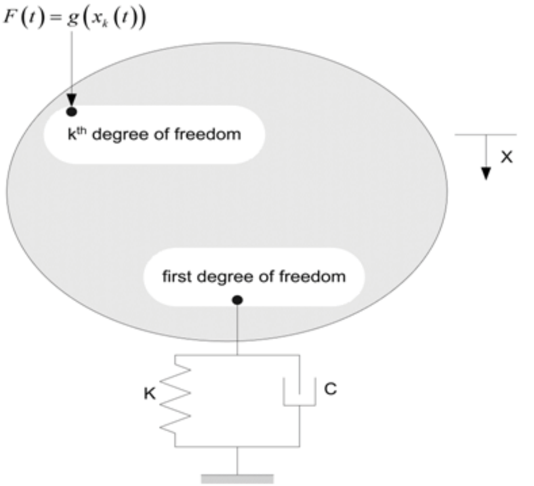

Fig. 4 Linear system subjected to a nonlinear force.

$$[m]_{nxn}\{\ddot{x}\}_{nx1}+[c]_{nxn}\{\dot{x}\}_{nx1}+[k]_{nxn}\{x\}_{nx1}=\{F\}_{nx1} \tag{4}$$

The vector $\{F\}$ includes the nonlinear forces. Using the same notation of Eq. The nonlinear force can be written:

$$\{F\}_{nx1}=-[A]_{nxn}\{g(x)\}_{nx1} \tag{5}$$

Matrix $[A]_{n\times n}$ is diagonal with non-zero elements only at the positions corresponding to the degrees of freedom where the nonlinear forces act. The same condition is verified for the vector $\{\mathrm{g}(\mathrm{x})\}$ where no null nonlinear functions exist only at each corresponding degree of freedom. Assuming that the system is connected to the inertial reference by a spring and damper, located at the degree of freedom “1”, representing for example, a load cell that connects this degree of freedom to the ground, Eq.4 and Eq.5 can be rewritten as follows.

$$\begin{gathered}[m]_{nxn}\{\ddot{x}\}_{nx1}+[c']_{nxn}\{\dot{x}\}_{nx1}+[k']_{nxn}\{x\}_{nx1}=-[A]_{nxn}\{g(x)\}_{nx1}+\{F_{load\_cell}\}\\ \{F_{load\ \ cell}\}=-[c'']_{nxn}\{\dot{x}\}_{nx1}-[k'']_{nxn}\{x\}_{nx1}\end{gathered} \tag{6}$$

Where:

$$\begin{gathered}[c']_{nxn}=[c]_{nxn}-[c'']_{nxn}\\ [k']_{nxn}=[k]_{nxn}-[k'']_{nxn}\\ [c'']_{nxn}=\begin{bmatrix}c_1 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & 0\end{bmatrix}_{nxn}\\ [k'']_{nxn}=\begin{bmatrix}k_1 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & 0\end{bmatrix}_{nxn}\end{gathered} \tag{7}$$

$$[m]_{nxn}\{\ddot{x}\}_{nx1}+[c']_{nxn}\{\dot{x}\}_{nx1}+[k']_{nxn}\{x\}_{nx1}+[A]_{nxn}\{g(x)\}_{nx1}=\{F_{load\_cell}\} \tag{8}$$

The frequency domain representation of the linear system, subjected to nonlinear forces, is finally obtained by taking the Fourier transform on both sides of Eq.

$$\{F_{load\_cell}(f)\}_{nx1}=[H'(f)]_{nxn}\{X(f)\}_{nx1}+[A(f)]_{nxn}\{\mathscr{F}(g(x_k))\}_{nx1} \tag{9}$$

Using Eq. and the concept presented in Fig. (1), the correct representation of a MISO system with nonlinear excitation force is constructed.

Fig. 5 MISO system used to indirectly determine the nonlinear force.

Matrix $[H]$ is replaced by matrix $[H']$ to represent the FRF of the system in Fig.(4) with the terms k_1 and c_1 removed from stiffness and damping matrix and transferred to the right side of Eq. Representing the force measured by the load cell. It should be noted that the force measured by the load cell includes the linear component $S_{y_h y_h}$ and the nonlinear component $S_{y_u y_u}$. Moreover, to obtain the true nonlinear force, by means of nonlinear MISO signal analysis, a prior knowledge of nonlinear function $g(x)$ is required.

As shown in Eq. (9) and in section 2 the force measured by the load cell can be completely separated in its nonlinear parcel and in its linear parcel. So the nonlinear excitation force is indirectly measured by means of the estimative of the nonlinear term in the Eq. (1) $S_{y_u y_u}$.

IV.NONLINEAR MODEL OF THE FRICTION FORCE

The proposed model for the friction force between two surfaces is capable to represent stick and slip displacements and also its hysteretic nature. The Coulomb friction law is used to relate the normal and tangential forces at the contact surfaces. Figure 6 presents the physical behavior of contact model between one static surface and a second body in one cycle of harmonic imposed displacement. The mathematical model is given on Eq. (10) for each kinematics condition at the contact.

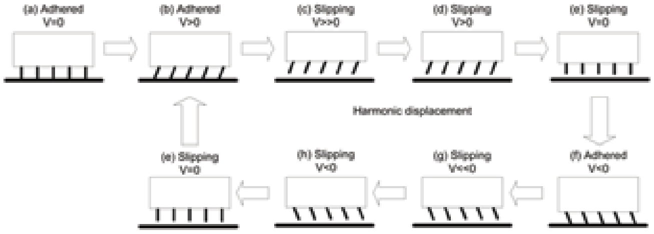

Fig. 6 Schematic diagram of the friction model.

if the body is at rest and will start the movement

$$F_{Coulomb} = \operatorname{sgn}(v_{rel}) * u_{rel} * K_{Coulomb}$$

if the body is slipping in the static surface

$$F_{Coulomb} = \operatorname{sgn}(v_{rel}) * F_{Coulomb}$$

$$u_s = u_{rel} \tag{10}$$

if the body is adhered to surface after its slip in the static surface

$$F_{Coulomb} = \operatorname{sgn}(v_{rel}) * (u_{rel} - u_s) * K_{Coulomb}$$

The slip displacement u_s do not change if the body is adhered to the static surface. Friction stiffness, related to elastic deformation of the asperities in contact, is defined as $K_{Coulomb}$. The relative velocity and displacement are defined as v_{rel} and u_{rel}, respectively.

Using this model, the nonlinear function $g(x)$ of the MISO analysis, is completely defined. It can be verified that $F_{Coulomb}$ is function only of the state variables of the global system. [3]

Imposing $u_{rel}(t)$ as a sinusoidal displacement on Eq., the normalized the friction force behavior is calculated and presented by Fig.7.

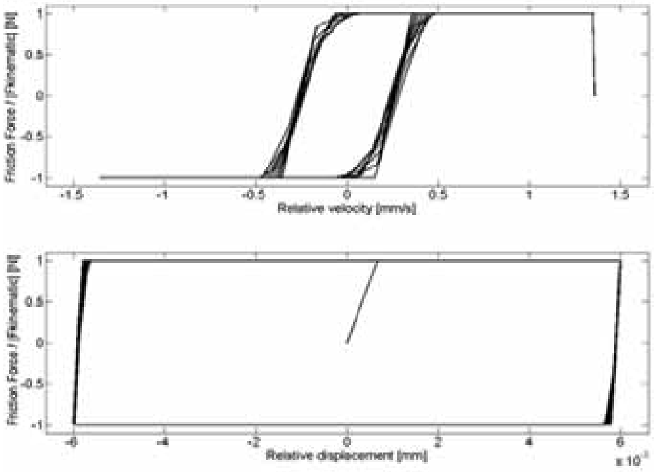

Fig.7 Behavior of nonlinear friction model.

The hysteretic and chaotic natures of the friction are evident. Comparing Fig.6 and Fig.7 should be noted that the elastic potential energy, stored in the deformed asperities in contact, was not released until the body and the surface adhere again. So this condition is different from the start of movement, when the adhesion between the body and the surface occurs without any stored potential energy in the contact. Hysteresis is represented by transition from stage “h” to stage “b” and from stage “d” to stage “f”, as shown in the Fig. 6.

V. RESULTS AND DISCUSSION

Application of the nonlinear MISO signal analysis in wear experiments is directly attained to the capacity of this methodology to provide the best estimate to friction force. Two classes of experiments were conducted to show the friction force identification capability.

A. One degree of freedom linear system with a Coulomb friction damper: In this group of experiments the vibratory system was excited by an harmonic force. Friction force due a Coulomb damper is obtained by MISO nonlinear signal analysis, then compared with the signal measured by a load cell placed between the mass and the Coulomb friction damper;

B. Reciprocating wear test in one specimen of aluminum covered by a small layer of Teflon™: This group of experiments was conducted to verify if the friction force estimated by MISO nonlinear signal analysis is sensitive to changes of the tribological conditions on wear tests.

The experiment, conducted with a linear vibratory system with a Coulomb damper attached, was used to confirm the validity of adopted law of friction. Since the linear parameters of the vibratory system are well known by traditional analysis the influence caused by Coulomb damper can be easily identified.

Second group of experiment was used to show the sensibility of estimated friction force in changes of tribological condition of the two bodies in contact. The experiment has the characteristic of small changes in the friction force amplitude with the wear of Teflon™ layer, with the increase of Teflon™ wear the friction force increase gradually until the Teflon™ layer was completely removed from the contact area. So this experiment is suitable to cover a wide range of changes in the amplitude of fiction force.

Case A: Coulomb damper on a one degree of freedom vibratory system.

The experimental apparatus used to analyze case A is shown in Fig. 8. A load cell was placed between the table and the Coulomb damper to measure the friction force. However, the signal measured by the load cell 02 is not the actual friction force, since it is the summation of friction force and inertia force generated by the moving mass of the Coulomb damper.

The nonlinear system represented in the Fig.3 is correlated to experimental apparatus as follow:

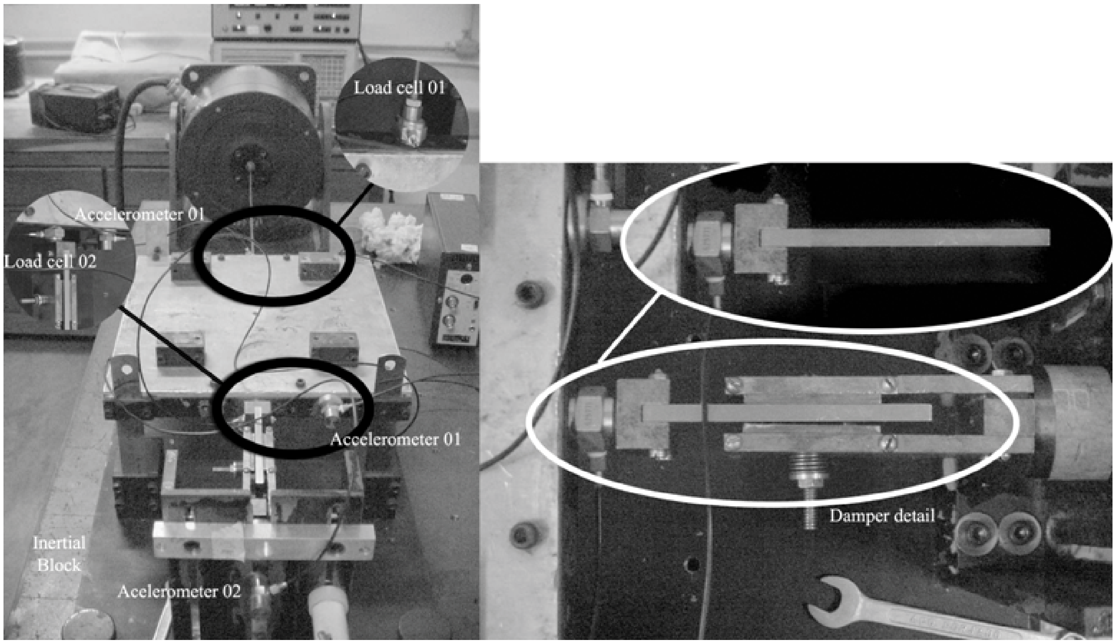

Fig. 8 Experimental apparatus used in group tests A.

- Linear system: Vibratory table supported by four thin bars connected in parallel, with one degree of freedom at the frequency band of interest;
- Nonlinear device: Coulomb damper installed between the table and a fixed 'rigid' support.

As shown in the Fig. 8 the table movement was measured by the accelerometer 01, and the accelerometer 02 measured the small displacement of damper 'rigid' support. Load cell 01 measured the excitation force and load cell 02 measured the force due Coulomb friction damper plus the inertia force of damper moving part. So, the nonlinear MISO system was configured as follows:

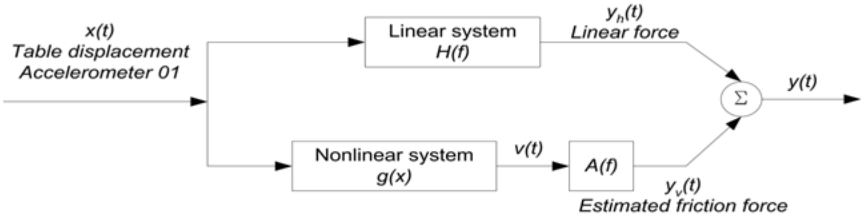

Fig. 9 Nonlinear system configuration to group tests A.

Experiments used to validate the nonlinear MISO formulation were conducted with harmonic excitation forces with frequencies 5 Hz up and 18 Hz. These frequencies were chosen to assure that the force measured by load cell 02 be the best approximation of the real friction force, minimizing the effect caused by the inertia force of the damper apparatus moving parts and the vibrations amplitudes of the fixed 'rigid' support. These frequencies were selected analyzing the cross correlation between the acceleration measured by accelerometer 02 with the force measured by the load cell 02, using a white noise force as excitation. This experiment was conducted when the Coulomb damper was mounted like in the damper detail of shown in the Fig. 8. The result is shown in the Fig. 10.

As should be noted in the Fig. 10, the frequencies above 20 Hz have a great influence of the vibratory system composed of the damper moving part and the load cell as a suspension. These influences can also be due to the bending effect in the load cell, small clearance of the joint parts and due small misalignment of the center of mass of moving parts with the load cell geometric center. Since inertia forces are directly proportional to acceleration, and it is proportional to the square of frequency, these influences in higher frequency agree with theory. Therefore, when frequency increases the friction force become very small than the other forces measured by the load cell 02. Additionally, the mass of the damper moving part was identified dynamically and statically and the results are 0.227 kg and 0.206 kg, respectively.

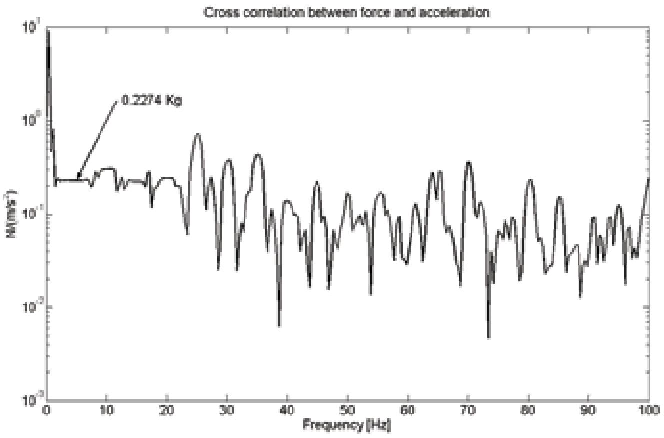

Fig.10 Cross correlation between the signals of accelerometer 01 and load cell 02

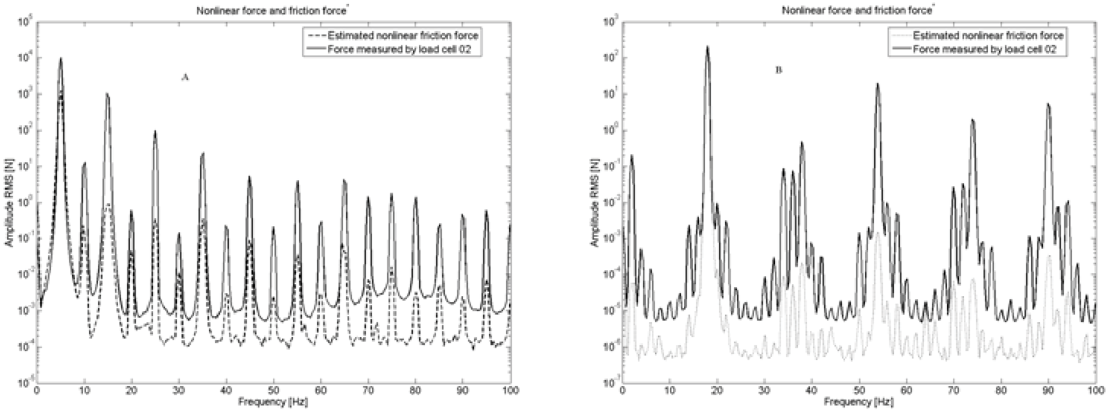

Fig.11 Results from group test A: (a) with 5 Hz and (b) with 18 Hz harmonic excitation forces.

The obtained results with harmonic forces are presented at the following figures: Fig.11, Fig.12a and Fig 12b for two excitation frequencies.

Should be noted in the Fig. 11 that at the excitation frequencies there is a small difference between the force measured by the load cell 02 and the estimated friction force due to the inertial force effect cited before. Another source of error in the estimated friction force is the nonlinear function $g(x)$ presented by Eq.. This source of error appears since the theoretical friction force is assumed to be square wave, with a small distortion due hysteretic effect, but do not represent other effects, which can exist in the real friction force or in the measured force. As can be seem in the Fig. 12, the inertia effect promotes the lobes of the measured force for velocity modulus values greater than 0.01 m/s.

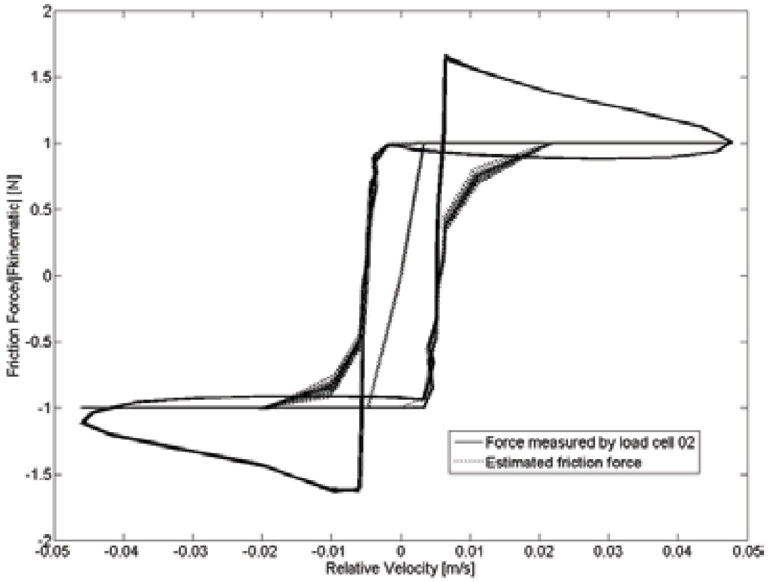

Fig. 12 Behavior of nonlinear friction model and force measured by load cell 02 from group test A, with 18 Hz harmonic excitation force.

However, the proposed model is capable to represent the main effects, including stick and slip phenomenon, and permits a good approximation to this type of force. Inertia effects are responsible by the greater difference present at higher harmonics of the excitation frequency, but they are extracted by the proposed methodology. An amplitude modulation effect is observed in Fig.11b, generated by the motion of the large inertial block, upon which the vibratory system is mounted, whose natural frequencies are about 2.5 Hz.

Case B: Reciprocating wear test in one specimen of aluminum covered by a thin Teflon™ layer.

Experiments with the reciprocating wear test machine were conducted using the formulation presented by Eq. And the nonlinear force model presented by Eq.. The diagram of the apparatus, the instrumentation chain and some of the test conditions are presented at Fig.13.

Based on Fig. 13 and Eq. the block diagram associated with the nonlinear MISO system is constructed and presented in Fig.14.

The reciprocating wear experiments take 2 hours to complete. The signals are acquired with 10 kHz rate using a simultaneously 16 channel AD converter with 12 bits of resolution. Averaging 10 blocks with 32768 points each, the results are stored in hard disk at 60 seconds interval. The tested specimens were analyzed in a scanning electron microscope (SEM) to compare the surface condition and the wear mechanism with the evolution of the friction force estimated by nonlinear MISO technique.

Three classes of experiments were carried out regarding the final surface condition of the aluminum specimen: I) the experiment was stopped before any damage occur to

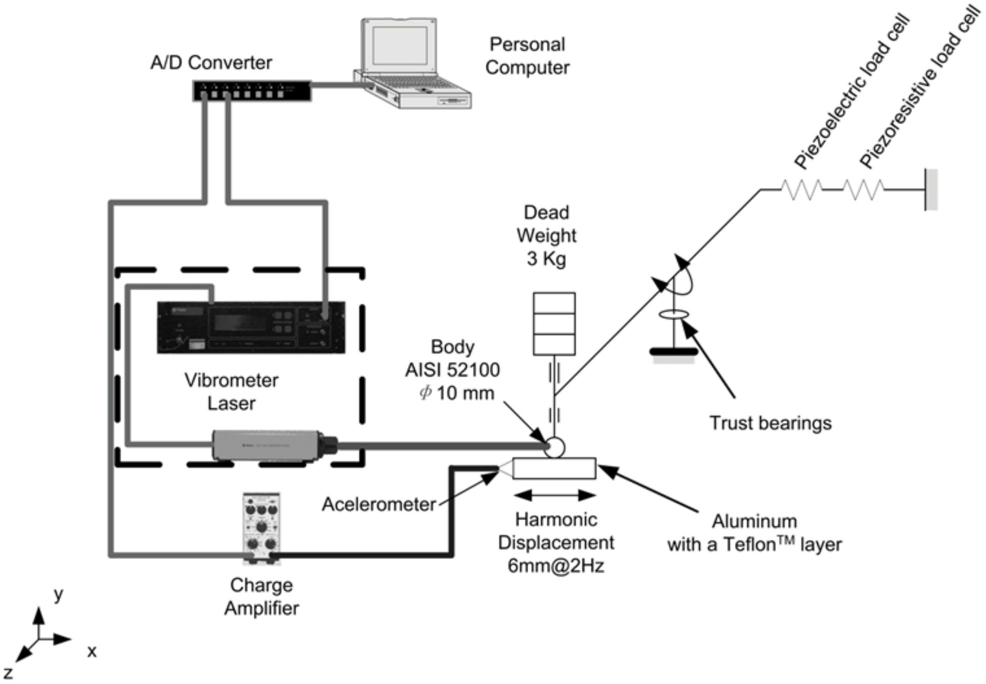

Fig. 13 Experimental apparatus and the instrumentation chain used in reciprocating wear tests.

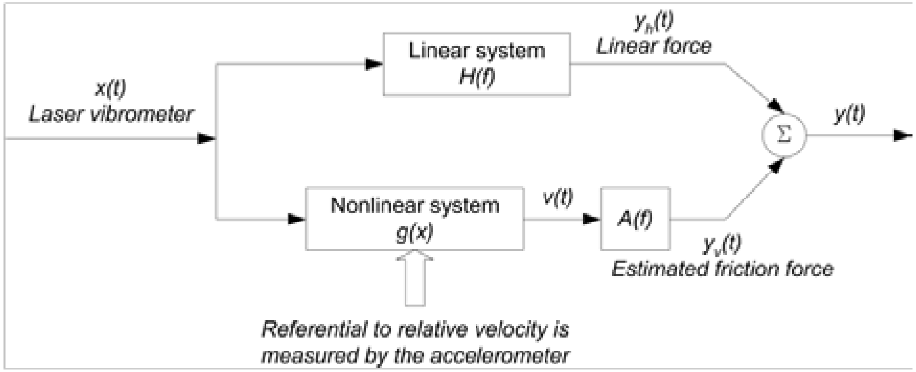

Fig. 14. Nonlinear MISO system used to the reciprocating wear tests.

the Teflon™ layer; II) the experiment was stopped when the layer degradation is minimum; III) the experiment was conducted until the Teflon™ layer was completely removed.

Results obtained with nonlinear analysis are presented by the RMS value of the estimated friction force in frequency band of 0-10Hz. This frequency band was choosing since this appear as the frequency band where the coherence function between the input data and output data assume the greater values.

Figure 15 shows that friction force has a small oscillation of its RMS amplitude, around the value of 0.23 N, as well as the changes in the surface condition along 1800 seconds of the experiment is small. However, Fig 16 and Fig. 17 show an increase on the mean square force amplitude, which is correlated to the surface conditions, indicated by the formation of some cracks on the Teflon™ layer, followed by its fragmentation promoted by the contact force. In these figures should be noted that RMS values change from 0.23 N to values between 0.6 N and 1 N.

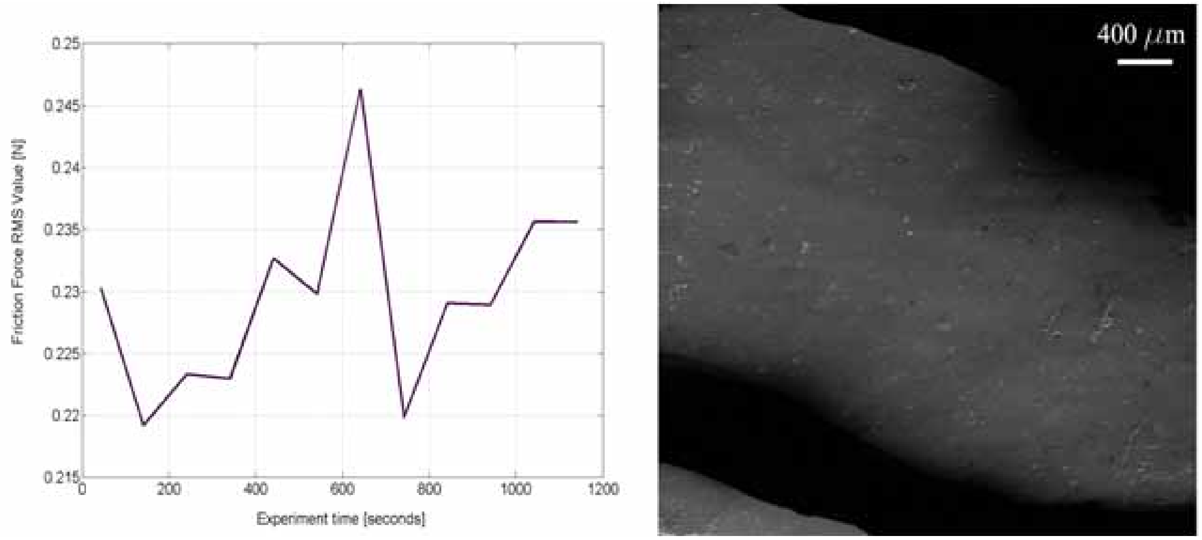

Fig. 15 Nonlinear MISO results and SEM images from Class I experiments.

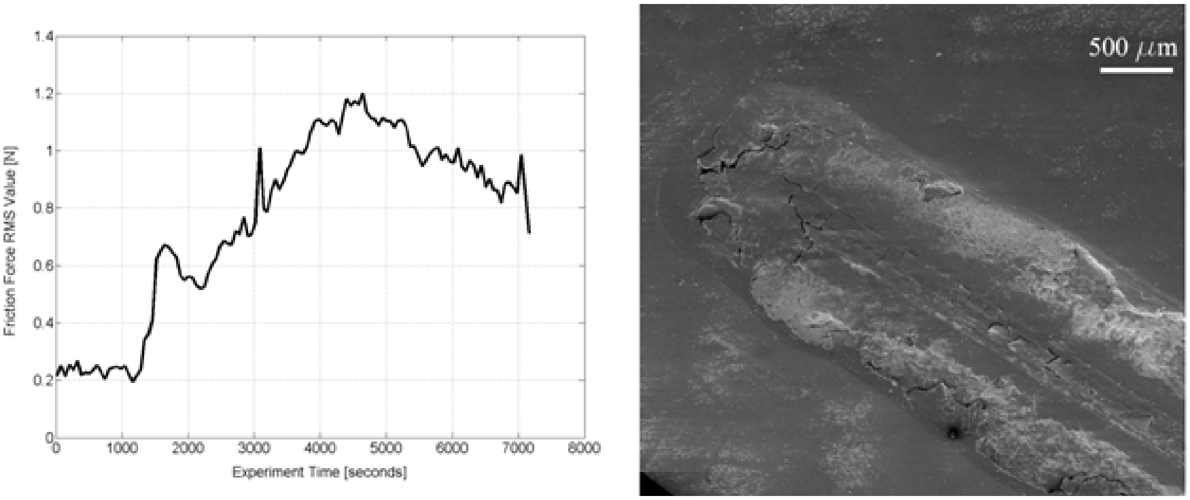

Fig.16 Nonlinear MISO results and SEM images from Class II experiments.

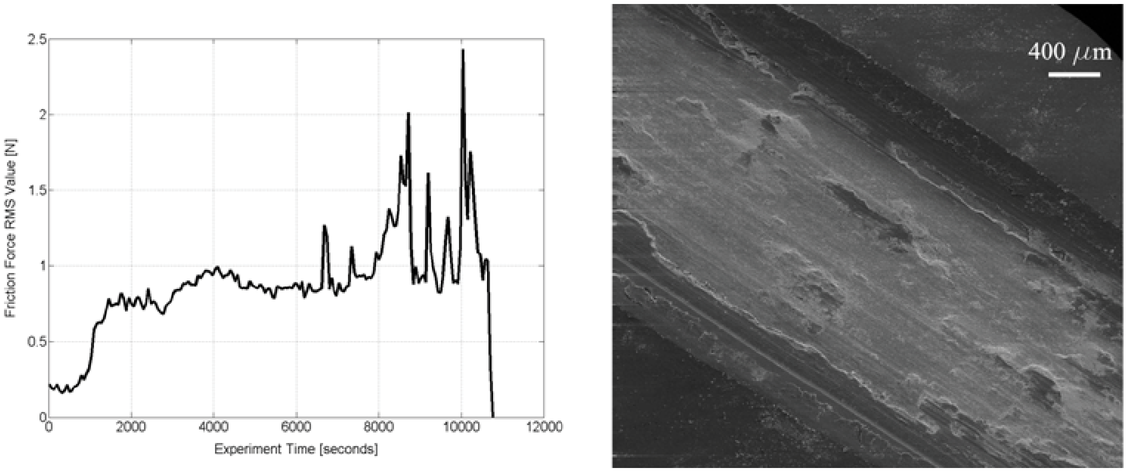

Fig.17 Nonlinear MISO results and SEM images from lass III experiments.

After fragmentation of the Teflon™ small particles remains in the contact region, until start its remotion, event that occur around 6000 seconds as shown in Fig.17. After 9000 seconds the RMS values of the estimated friction force in this frequency band presents some spikes that seems to be related with contact between the sphere and the aluminum from the substrate of the prismatic body indicating the completely remotion of Teflon™.

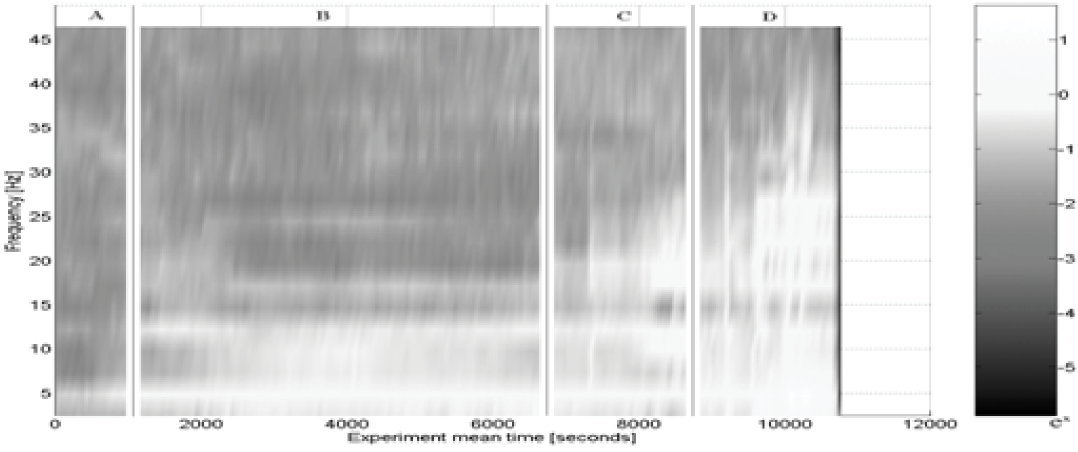

Fig. 18 Frequency band from 0 to 100 Hz to experiment group B class III.

Figure 18 shows four stages in the experiment where the Teflon™ layer was completely removed. These stages are characterized by the increase of the friction force mean square values. Should be noted the correlation of estimated friction force with wear evolution. Initially no damage appear at Teflon™ layer, region "A" in Fig. 18. The action of counter body produces some cracks and fragments this cracked layer, region "B" in Fig. 18, after this the relative displacement removes the Teflon™ powder, region "C", until the contact specimen area is only aluminum and the friction force RMS level reaches its maximum in the region "D" of the Fig. 18.

VI. CONCLUSION

Analysis and interpretation of nonlinear behavior that occurs in linear systems is feasible by the usage of nonlinear MISO technique. The proposed methodology is useful to identify nonlinear effects that act on linear system without restrictions on the statistical nature of the measured signals. If the nonlinear behavior is well modeled, the proposed methodology is capable to identify and separate the exact nonlinear part from the global response of the system.

In case of mechanical systems with known linear properties the MISO representation is simple and easy to be physically interpreted than by other nonlinear analysis techniques. Since the methodology does not impose any restrictions on the nature of nonlinear functions, it finds usefulness in the identification of physical parameters of highly nonlinear mechanical systems.

A methodology to measure the friction force in the vibratory systems upon its response was presented. This methodology is capable to estimate the friction force without the influence of the system dynamics. The proposed methodology provides an estimative of friction force that is sensitive to changes in tribological conditions of a surface under wear tests. This characteristic is useful to identify specimens surface wear conditions as such the time of occurrence of discrete events, as observed on the experiment with bisulphate of molybdenum thin layer. The nonlinear MISO shows that it is possible to tribology researchers to study alternative mathematical formulations to represent friction between bodies. Since the friction force is identified, static and kinetic friction coefficients can be calculated if the normal forces between two bodies are measured.

The proposed methodology permits the identification of surface tribologic phenomena by analyzing the signals at higher or lower frequency bands. The best band may be selected depending on the correlation of the identified signal with the surface characteristics measured, for example, on a scanning electronic microscope. The frequency band near the frequency of the imposed harmonic displacement carries information about the friction force as expected by the adoption of the Coulomb law of friction. The proposed nonlinear friction model permits the experimentally identification of the contact stiffness. This characteristic is useful to find tribological properties of bodies in contact, as in the study of solid lubricants performance.

REFERENCES

[1] Bendat, J. S., Palo, P. A. and Coppolino, R. N., A general indentification technique for non-linear differential equations of motions, *Probabilist Engineering Mechanics,* 1992, Vol. 7, pp. 43-61.

[2] Bendat, J. S., Nonlinear Systems Techiniques and Applications, John Wiley and Sons Inc., New York, 1997.

[3] Lépore Neto, F. P.; Mello, J. D. B. , Santos, M. B.,Correlação entre a força de atrito obtida através de análise de sinais não linear e o desgaste em um tribômetro do tipo alternativo., In: 59 Annual Congress – ABM International, 2004, São Paulo. 59 Annual Congress - International. São Paulo, ABM Brasil, 2004. pp. 4434-4443.

[4] Nowotny, S. L. R. H. and Wayne, S. F., The role of specimen stiffness in sliding and impact wear, *Wear,* Volume 77, Issue 1, 15 March 1982, Pages 13-28.