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Using Fuzzy Bang-Bang Relay Controller for a Single-Axis Magnetic Bearing System

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***Abstract* – This paper presents a new type of fuzzy controller for active magnetic bearing applications. Active magnetic bearing (AMB) applications in rotating machinery are fast growing due to their precise and contact less support of the rotating shaft. AMB are open loop unstable due to nonlinear relationship between electromagnetic force, attraction distance and the electromagnetic current. To regulate the electromagnetic forces acting on the bearing, external control is required. Feedback control for AMB systems such as proportional and derivative is only restricted to linearized region. For nonlinear control systems, artificial intelligence techniques such as fuzzy and hybrid techniques are being investigated. Bang-bang control is an old but effective technique to control nonlinear system in optimal time. Bang-bang control combined with fuzzy logic decision-making flexibility results in a robust control system. In this work an integrated fuzzy bang-bang relay controller (FBBRC) is presented to control the AMB system. FBBRC is simple to design than conventional fuzzy controllers. Comparison with other widely used AMB control techniques demonstrate improved results.**

***Keywords:* Bang-bang fuzzy logic, Magnetic bearing, Nonlinear control, Proportional derivative, Electromagnetic force**

I. Introduction

The utilization of active magnetic bearings in rotating machinery installations has seen a steady increase over the years. This is mainly due to their advantages over the conventional fluid-film and rolling-element bearing types for some specific applications. Amongst the major advantages of these bearing types are their oil-free and contact less operation, which eliminates the need for lubrication and avoids the problem of wear. The magnetic bearings, however, exhibit highly nonlinear characteristics due to the relationship between the forces generated in their actuators with the coil current and the air gap between the rotor and the stator. These bearings are open-loop unstable but they can be easily stabilized using feedback control strategies, of which the most widely used in practical applications is the linear PID controller [1]. The linear PID controller is however only effective when the magnetic bearing operates at the vicinity of the equilibrium point where its dynamics are linearized. Its performance may rapidly deteriorate as the operation of the bearing deviates from this equilibrium point. Various control strategies have been proposed to compensate for the nonlinear dynamics of these bearings so that their operating region can be extended. Several authors have investigated fuzzy logic and optimal time bang-bang control schemes in the past. A brief review of the applications of these schemes to the control of magnetic bearings follows.

Liebert [2] proposed an adaptive fuzzy scheme for the control of active magnetic bearings. This scheme changes the linear PID controller gain by using fuzzy logic. Measurements performed on a magnetic bearing test rig showed that this control scheme significantly improved the step response as compared to the linear PID and steady-state controllers. Hung [3] proposed a variable structure scheme for the control of a single-axis magnetic bearing system. This scheme utilized a linear PID controller when the bearing was operating near the equilibrium point, and for operation away from this point a nonlinear feedback-linearizing controller was used instead. A fuzzy controller was developed to provide a smooth transition between these two control structures and to avoid chattering in the response of the magnetic bearing system. A control scheme based on the Takagi-Sugeno-Kang fuzzy model, proposed by Hong *et al*. [4] for the control of a nonlinear magnetic bearing system, demonstrated performance that was superior to the single operating point linear controller. Numerical simulation results also showed that this control scheme maximized the stable region of operation of the magnetic bearing system. This effectiveness of this control scheme was further verified experimentally on a two-axis vertical rotor-magnetic bearing system with symmetric structure [5]. Kosaki *et al*. [6] proposed a multivariable fuzzy controller for a magnetic bearing system with gyroscopic cross-coupling effect and numerically demonstrated its capability to maintain the response performance of the system when subjected to external disturbance forces and rotor speed variation. Numerical results on the control of prescribed motion using a magnetic bearing in an active rotating stall control test rig were presented by Lei *et al*. [7]. The magnetic bearing, whose main function was to partly support the test rig, was also used as an exciter to generate rotor whirl orbits of specified magnitudes. The fuzzy logic controller developed in this work was able to increase the rotor whirl orbit radius by fifty percent of what was achievable using a conventional PD controller.

Most of the early work on AMB controller design was focused on extending their operating range. Fuzzy controllers alone, or working in tandem with PID are robust solution to the nonlinear dynamic problem. Fuzzy controller developed in past for AMB does not focus on minimizing the response time. Minimum time response of the controller is its ability to respond in shortest possible time and is required for fast switching action for AMB system. The response of fuzzy controller depends upon its fuzzy set, which includes input and output membership functions rules, their partitioning and overlapping. The fuzzy sets are imprecise and are chosen on *ad hoc* basis. In absence of any guidelines to set the fuzzy controllers, they are tuned to meet the specific response by optimization or neural networks training techniques.

A robust nonlinear control method known as 'Sliding mode control' (SMC) has been used for AMB control. This method tolerates the variability and un-modeled dynamics of the plant. Numerous researchers have applied sliding mode control to AMB. Maslen E and Montie D.[2001] [8] argued that sliding mode control offered no advantage over conventional PID control of AMB. Allaire and Sinha [9] used sliding mode techniques to develop controllers robust to several potentials sources of uncertainty in the plant. And Tian *et al* [10] looked at discrete sliding mode control of flexible rotors including a disturbance model. The main idea behind sliding mode control [11] is to convert the control problem for multi-state control systems to a simpler first-order control problem. SMC provides a satisfactory performance with a simplest control structure but comes with undesired high control activity at steady state and slow switching time.

Foust H. *et al* [12] demonstrated that sliding mode control (SMC) of first order system is equivalent to Bang-bang control. To obtain time optimal condition the SMC directs the control activity through bang-bang controllers, which are entrusted for minimum time response. The Bang-bang control, which switches between extreme opposites, yields minimum-time control of the system [13]. The Pontrygin minimum Principle (PMP) has been extensively used to design time optimal control [14]. PMP states that Hamiltonian function described by states and costate trajectories together with control effort in minimum time, when solved, yield the optimal state trajectory corresponding to optimal control effort. It is not a surprise that the sliding line of SMC and state trajectory solution of PMP optimal control have almost similar control laws as shown by Kulczycki [15], thus establishing the fact that fuzzy bang-bang control is indeed a robust control system. The combination of fuzzy and bang-bang control offers a robust controller, which is capable of controlling nonlinear system in minimum time. One of the earliest fuzzy bang-bang controllers (FBBC) was developed by Chiang and Jang [16]. It made its debut in Cassini spacecraft's deep space exploration project. The controller proves its superiority over the conventional bang-bang controller. Other applications include minimum time fuzzy satellite attitude controller [17], crane hoisting and lowering operation [18] and in process control valves operation [19].

Conventional bang-bang controllers are made from electromechanical relays that are getting obsolete owing to the fact that their parameters are fixed and act slowly.

Solid-state relays are fast-acting but are not flexible to control nonlinear systems over the entire operating range. The demand for flexible and programmable relays has grown in recent years. In this paper a new integrated configuration of a fuzzy controller is proposed. This controller directly produces two-level bang-bang crisp output, which is based on the Largest of Maxima (LOM) defuzzification method. The proposed controller does not require any further saturation or hard limiting device. The consequent part of the fuzzy rules has only two linguistic values, while the premise parts are freely chosen. The proposed fuzzy bang-bang relay controller (FBBRC) works exactly like a two-level relay and has flexible output to control nonlinear system. It is structurally simple due to two membership function in its fuzzy output set and rule matrix.

This paper is outlined as follows. In section 2, a single axis active magnetic bearing model is presented for development of FBBRC. In section 3, FBBRC is developed with a fuzzy logic set structure and comparison is made between four different controllers that is FBBRC, conventional PD, standard fuzzy logic controller (FLC) with and without the hard limiter device. In section 4, the controllers' stability and optimality are analyzed. Finally, the paper is concluded in section 5 with future work propositions.

II. Single-Axis Magnetic Bearing System Model

A magnetic bearing actuator system consists of a stator and a rotor. A magnetic field is created within the stator, rotor, and the air gap between the stator and the rotor when current flows in the coils that are wound around the stator. The magnetic actuator force, which is based on the field energy in the air gap, is given in the following equation, where k is the force constant that is a function of permeability of free space μ_0, air gap cross-sectional area A_g and number of coil turns N (Schweitzer *et al*. 1994) [20].

$$F = k\frac{i^2}{4g_0^2} \tag{1}$$

In the actual operation of a magnetic bearing system, a pair of magnetic actuators counter-acting each other is used. This configuration, which is known as the differential driving mode and shown in Figure 1, allows both positive and negative forces to be generated. In this mode of operation, one magnetic actuator is driven with the sum of bias current and perturbation current ($i_b + i_p$), whilst the opposite one with the difference of bias current and perturbation current ($i_b - i_p$). In order to obtain the maximum dynamic range of the actuator, the bias current i_b is usually set to approximately half the saturation current. The net force produced by the counter-acting arrangement of the magnetic actuators shown in Figure 1.

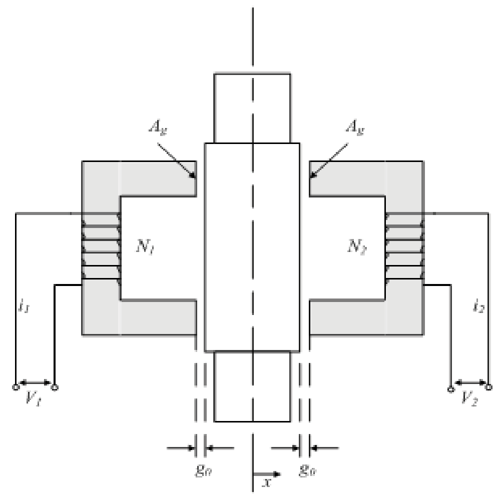

Fig.1 Schematic of a single-axis magnetic bearing actuator system

The air gap is reduced by ($g_0 - x$) in the decreasing side, and increased by ($g_0 + x$) in the opposite side. i_x is the perturbation current for the actuator in the X -axis, g_0 is the nominal gap, i.e., the gap at equilibrium position where the perturbation current is zero, and x is the displacement of the rotor from its equilibrium position in the X -direction. F_1 And F_2 denote the forces in the two counter-acting actuators, respectively, whilst F_x denotes the resultant force in the X -direction. Substituting these values into Equation (1) gives the non-linear force equation for the counter-acting actuator system.

$$F_x = F_1 - F_2 = k\left[\frac{(i_b + i_x)^2}{(g_0 - x)^2} - \frac{(i_b - i_x)^2}{(g_0 + x)^2}\right] \tag{2}$$

Applying Newton's second law to the single-axis magnetic bearing system shown in Figure 1 yield the following governing equation, where m is the mass of the rotor.

$$m\ddot{x} = F_x = k\left[\frac{(i_b + i_c)^2}{(g_0 - x)^2} - \frac{(i_b - i_c)^2}{(g_0 + x)^2}\right] \tag{3}$$

The mathematical model given in Equation (3) is graphically modeled in figure 2 and is used in the numerical simulation. The block *f(u)* housed the nonlinear actuation force F_x, given in Equation (2). Also shown in figure 2, is the FBBRC, the proposed controller along with conventional PD and Standard fuzzy logic controller (FLC) with and without hard limiter. The controllers in figure 2 are compared, given the similar initial conditions. The details of the controllers are discussed in next section. The system parameters are based on the work of Hung (1995) [3], and are reproduced in Table 1.

TABLE I SYSTEM PARAMETERS

Mass of rotor, *m*	1 kg
Force constant, *k*	0.0001 Nm^2/A^2
Nominal air gap length, g_o	0.001 m
Bias magnetic coil current, i_b	0.5 A

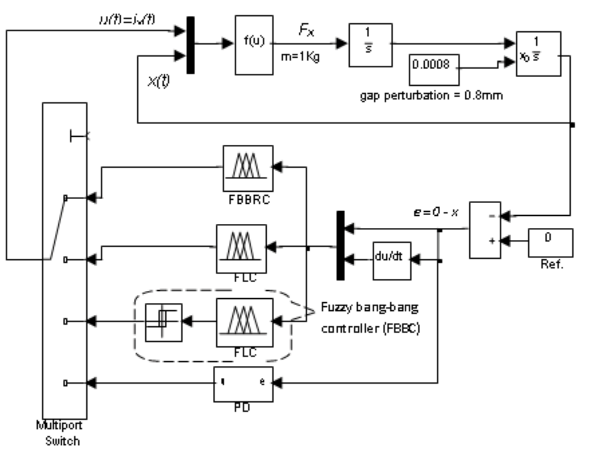

Fig. 2 Four controllers for one axis active magnetic bearing system: Multi-port switch is used to select the controller

III. FUZZY CONTROLLER DESIGN

Two types of fuzzy controller are described in this section. First, the new proposed controller, which combines the fuzzy logic with a hard limiter relay into one entity, is presented. This controller is defined as fuzzy bang-bang relay controller (FBBRC). Second, the conventional fuzzy logic controller (FLC) is presented for the comparison purpose. Both controller uses similar inputs fuzzy set. However, the output fuzzy sets are different. The FBBRC takes advantage of the largest of maxima (LOM) defuzzification technique to yield a bang-bang output. The FLC uses the centroid defuzzification technique, and if cascade with hard limit relay, it constitutes fuzzy bang-bang controller (FBBC) [15-19].

A. Controller Fuzzy Set

For any fuzzy controller, it is necessary to determine the ranges of its input and output variables, which are considered to be a reasonable representation of all the situations that the controller may face and yield to stability and optimality conditions.

B. Linguistic Ranges

Based upon the operational details of the model describe in Figure 1 and table 1, the air gap, g_0 of AMB is 1mm. The displacement $x(t)$ is referenced from the equilibrium position $x(t)$ = 0mm in the X -direction, set range of $x(t)$ with the universe of discourse of $X_1 \in$ [-1, 1]mm. To find the range of $\dot{x}(t)$, the simulation in figure 2 is run in open loop by setting $i_x(0) = 0$, with initial displacement of $x(0)$ = 0.8mm. The system driving force in open loop is owing to $i_b(0)$ = 0.5A, as given in table1. The open loop response is shown in figure3. The initial force F_o shown in figure 3a is evaluated as

$$\begin{aligned} F_x &= k\left[\frac{(i_b+i_x)^2}{(g_0-x)^2}-\frac{(i_b-i_x)^2}{(g_0+x)^2}\right] \\ F_o &= 10^{-4}\left[\frac{(0.5+0)^2}{(1-0.8)^2}-\frac{(0.5-0)^2}{(1+0.8)^2}\right] \\ &= 618N \end{aligned} \tag{4}$$

Since Mass = 1kg, the initial force F_o in Equation (4) is same as initial acceleration. The integration of Equation (4) as per simulation diagram, figure 2, is shown in figure 3b.The maximum velocity attainable by the rotor is $\dot{x}(t)$ ≈ 3.9m/sec. The displacement of the rotor $x(t)$ with initial position $x(0)$=0.8mm is shown in figure 3c.

The velocity curve in Figure 3b breaks sharply at 1 m/sec, setting the limit of $\dot{x}(t)$ to universe of discourse $X_2 \in$ [-1 ,1] m/sec. The sharp discontinuity occurring at $x(t) = g_o$ is noticeable from Equation (2).

To operate AMB with bang-bang action, two extremes of output level are required. The bang-bang action is produced by switching F_1 or F_2 on/off (never both at same time) to balance the rotor in equilibrium position, $x(t)$ = 0. The

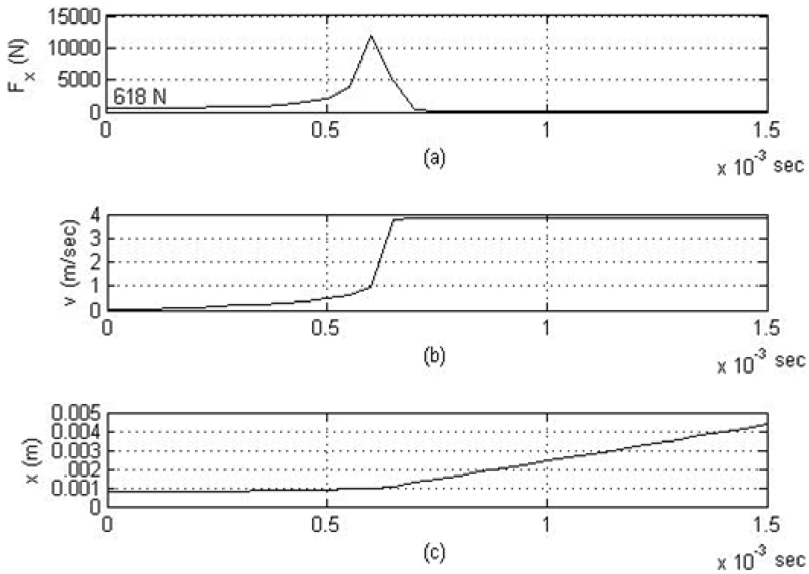

Fig.3 Open loop response of Equation (2) for initial position of x_o = 0.8mm and bias current i_b= 0.5A (a) The force response curve, initial force, $F_o \approx$ 618 N. (b) Displacement velocity curve of the rotor, maximum velocity ≈3.9 m/sec. (c) Displacement of rotor from the initial position

control law decides the choice of applied force, and will be discussed later. The actuation of F_1 or F_2 is achieved from Equation (2) by switching the current $i_x(t)$ on/off. Since $i_b(t)$ = 0.5 A then switching $\text{sgn}(i_x(t)) = i_b(t) \equiv 0.5$A, produces the bang-bang action, thus setting the output universe of discourse $Y_{bb} = [-i_x(t), +i_x(t)]$ A, for the bang-bang controller.

The set $\widetilde{A}_k^j$ defines the j^{th} linguistic value of k^{th} linguistic input variables $\widetilde{x}_{k=1}$ = 'error position' and $\widetilde{x}_{k=2}$ = 'error velocity', which in turn is defined over the universe of discourse $\boldsymbol{X}_k$. The control level of the system operation can be adequately defined for input $\widetilde{x}_1$ by the following $\widetilde{A}_{k=1}^j$, linguistic values:

$$\widetilde{A}_{k=1}^j = [\widetilde{A}_1^1 = LN,\ \widetilde{A}_1^2 = SN,\ \widetilde{A}_1^3 = Z,\ \widetilde{A}_1^4 = SP,\ \widetilde{A}_1^5 = LP]$$

Similar linguistic values are selected for input $\widetilde{x}_2$, $\widetilde{A}_2^j$ $\widetilde{A}_1^j$.

The set $\widetilde{B}_1^j$ denotes the linguistic values for FBBRC's output linguistic variable $\tilde{y}_1$, is defined as:

$$\widetilde{B}_k^j = \left[\widetilde{B}_1^1 \rightarrow I2,\ \widetilde{B}_1^2 \rightarrow I1\right]$$

Where, $I1 = +i_x(t)$ and $I2 = -i_x(t)$ are the on/off-firing command for actuation force F_1 and F_2 respectively.

C. Fuzzy Rules

The fuzzy linguistic rules assembled in this work reset the rotor to equilibrium position $x(t)$ = 0 mm. These rules are based on two input variables, each with five linguistic values, thus there are at most 25 possible rules. These rules are described in matrix form in Tables II and III. The main diagonal entry in the rules given in Table II is not used. The rules-partitions are heuristically chosen to balance the rotor smoothly over the universe of discourses.

Table II Fuzzy Rules For Fbbrc

$\dot{x}$ \ x	LN	SN	Z	SP	LP
LN	$+i_x$	$+i_x$	$+i_x$	$+i_x$	
SN	$+i_x$	$+i_x$	$+i_x$		$-i_x$
Z	$+i_x$	$+i_x$		$-i_x$	$-i_x$
SP	$+i_x$		$-i_x$	$-i_x$	$-i_x$
LP		$-i_x$	$-i_x$	$-i_x$	$-i_x$

TABLE III FUZZY RULES FOR CONVENTIONAL FLC

x / $\dot{x}$	LN	SN	Z	SP	LP
LN	PL*	PL	PL	PS	OFF
SN	PL	PL	PS	OFF	NS
Z	PL	PS	OFF	NS	NL
SP	PS	OFF	NS	NL	NL
LP	OFF	NS	NL	NL	NL

*PL= positive large, PS=positive small, NL= negative large

The symmetry of the rules matrix is expected as it arises from the symmetry of the system dynamics. The decomposition of linguistic rules from the FBBRC's inputs to the output is given by

$$\mu_{\tilde{B}^{j}_{k_c}}(y)=\min\left\{\mu_{\tilde{A}^{j}_{1_c}}(x_1),\ \mu_{\tilde{A}^{j}_{2_c}}(x_2)\right\} \tag{5}$$

The index c refers to the number of rules used in implication.

The conventional FLC uses the standard decomposition and centroid defuzzification techniques [21].

D. Fuzzy Set Membership Functions

The input linguistic variables and values assigned to fuzzy set membership functions are shown in Figures 4 and 5. Triangular shape membership functions are used in this work. These membership functions are sensitive to small changes that occur in the vicinity of their centers. A small change across the central membership function $\tilde{A}^3_k$, located at the origin, can produce abrupt switching of control command *u* between the *+ve* and *–ve* halves of the universe of discourse, resulting in chattering. The overlapping of the central membership functions $\tilde{A}^3_1$ with its neighboring membership functions $\tilde{A}^2_1$ and $\tilde{A}^4_1$ reduce the sensitivity of the bang-bang control action. Smooth transition between the adjacent membership functions is achieved with higher percentage of overlap, which is commonly set to 50%.

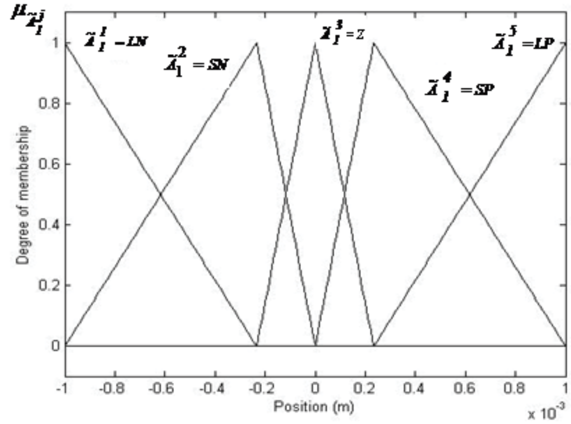

Fig.4 Membership functions of input $x \rightarrow$® $\tilde{x}_1$ = "error position" and linguistic values $\tilde{A}^j_1$ for both FBBRC and standard FLC

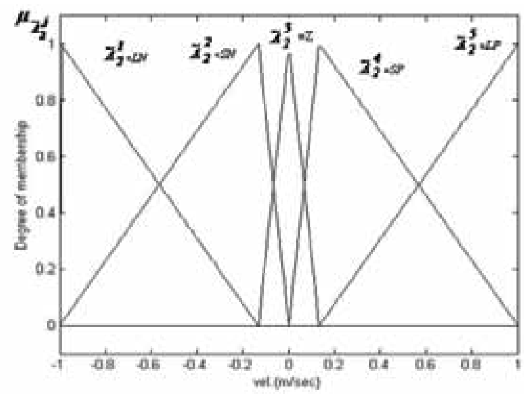

Fig.5 Membership functions of input $\dot{x}(t) \rightarrow \tilde{x}_2$ = "error velocity" and linguistic values $\tilde{A}^j_2$ for both FBBRC and FLC controller

The output membership functions for conventional FLC are shown in Figure 6 for Y_{FLC} ϵ [*-0.75*, *+0.75*] and for FBBRC in Figure 7. FBBRC has only two membership functions and there is no third central membership function at the origin of the output universe of discourse, as shown in Figure 7. As a result, there are no diagonal rules in Table II. For comparison purposes, the standard FLC (centroid output) and FBBRC use the same input membership functions as shown in Figures 4 and 5.

E. Largest of Maximum (LOM) Aggregation

The output membership functions shown in Figure 7a and the LOM aggregation, together formulates the fuzzy bang-bang relay controller. Any perturbation of the rotor from the equilibrium position acts on the output membership functions according to the rule matrix in Table2. The overall output of FBBRC depends upon the maximum value of degree of membership function, $\mu_{\tilde{B}^j_k}$ shown in Figure

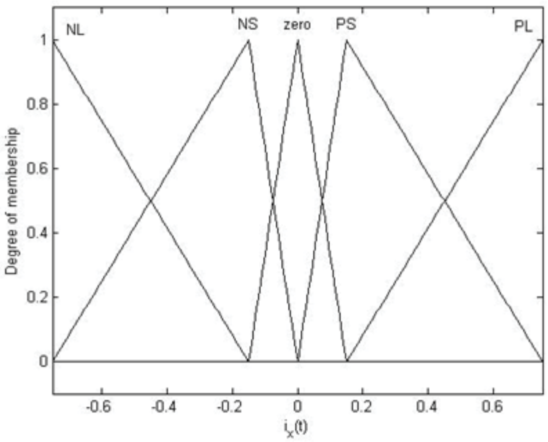

Fig.6 Output membership functions of conventional FLC gives centroid defuzzification output

6a. Denoting $\mu_{overall}$ as membership function of the overall implied fuzzy rules c =1, 2...*C*, is obtained by taking the maximum of aggregation described as

$$\mu_{overall}(y)=\max_{c}\left\{\mu_{\tilde{B}^{1}_{1_c}}(y),\mu_{\tilde{B}^{2}_{2_c}}(y),...\mu_{\tilde{B}^{j}_{k_c}}(y)\right\} \tag{6}$$

The defuzzified crisp output y^{crisp} based on Equation (3) can be evaluated as

$$y^{crisp}=\arg\sup\{\mu_{overall}(y)\} \tag{7}$$

The supremum in Equation (7) is the Largest of Maximum (LOM) value and occurs at the extremes of the output universe of discourse $\psi = [-i_x, i_x]$. The argument *arg (sup (μ))* returns ycrisp = [*-0.5, 0.5*]. The bang-bang firing action of membership functions *I2* and *I1* is shown in Figure 7b.

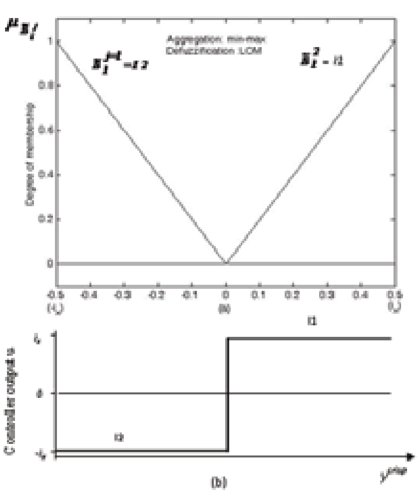

Fig.7 (a) FBBRC output *y* membership functions and linguistic values $\widetilde{B}_I^j$ (b) FBBRC two level Bang-Bang y^{crisp} output

F. Bang-bang Controllers Stability and Optimality

In the case of fuzzy bang-bang controllers, the heuristic approach of fuzzy rules result in partitioning of decisions space (phase plane) into two semi-planes by means of a *sliding (switching) line*. Similarity between fuzzy bang-bang controller and sliding mode controller (SMC) can be used to redefine the diagonal form of fuzzy logic controller (FLC) in terms of an SMC, with boundary limits, to verify the stability of the proposed bang-bang controller [22, 23]. SMC is a robust control method [24] and its stability is proven with lyapunov's direct method. So in lieu of SMC, the fuzzy bang-bang control stability can be easily established.

The Pontrygin's minimum Principle (PMP) has been used to design time optimal control for AMB application [25]. PMP states that Hamiltonian function described by states and costate trajectories together with control effort in minimum time, when solved, yield the optimal state trajectory corresponding to optimal control effort [26]. It is not a surprise that the sliding line of SMC and state trajectory solution of PMP optimal control have almost similar control laws as shown by Kulczycki [15], thus establishing the fact that fuzzy bang-bang control is indeed a robust control system.

G. Proportional Derivative controller

For comparison purpose a PD controller is used. The gains Kp and Kd for the controller are optimized as shown in the figure 8. The AMB system in Equation (3) is simulated from the initial rotor position $x(0)$=0.8mm to equilibrium position $x(t)$=0mm. Optimization procedure searches for Kp and Kd values inside the PD controller block to reset the rotor to equilibrium position. The returned optimized values are Kp=706.42 and Kp=1.03.

IV. Simulation Results and Discussion

Simulation results of the four controllers shown in figure 2 are analyzed in this section. The time responses are shown in Figure 9 and 10. The initial distance of the rotor is 0.8mm from the center. The settling time of FBBRC and FBBC is in order of 15msec and smaller than PD controller and conventional FLC. PD has larger overshoot than FLC.

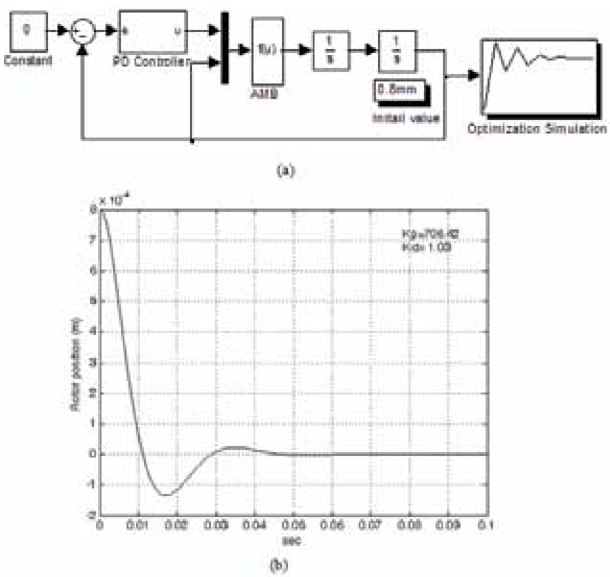

Fig.8. a Proportional derivative gain optimization simulation (b) Optimized output response for Rotor position $x(0)$= 0.8mm

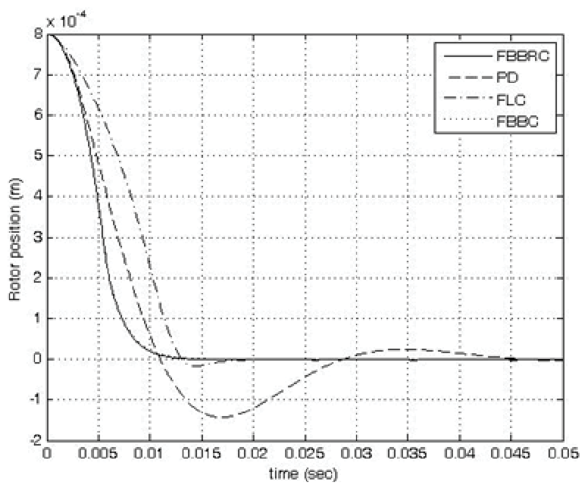

Fig.9 Time Response of controllers for initial position 0.8mm

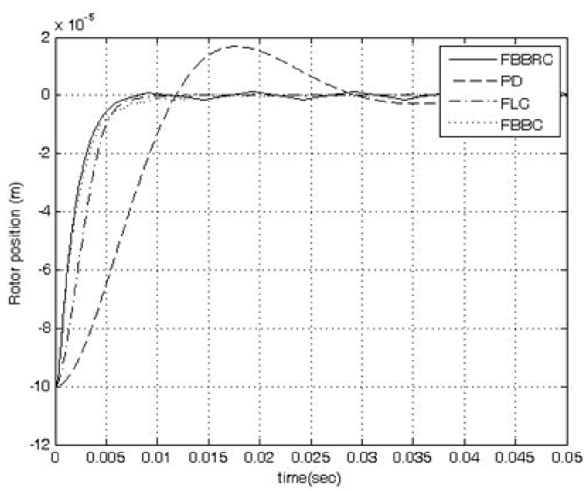

Fig.10 Response time from initial position of –0.1mm

The steady state response of FBBC and FBBRC are magnified in Figure 11. The steady state response reveals that FBBC has high frequency chattering in comparison to FBBRC. The high frequency can cause un-stability and distortion in the current amplifers.

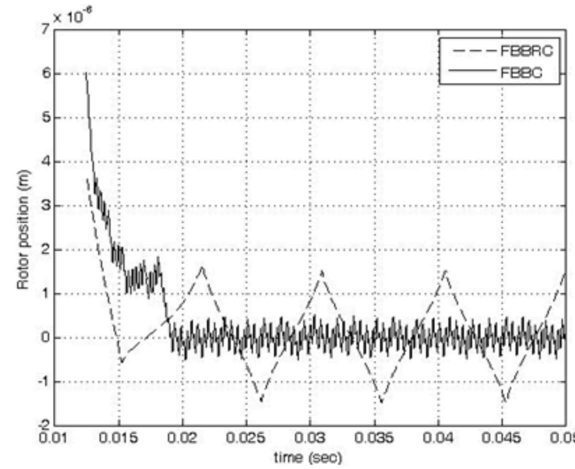

Fig.11 Steady state responses of FBBC and FBBRC. FBBC has high frequency chattering

The high frequency chattering arises from the relay output control signal as shown in figure 12. The low frequency output from FBBRC is in order of 100Hz while FBBC chattering is in order of KHz. For steady state, at average equilibrium position $\bar{x}=0$, the bang-bang output force $F_1 = sgn(F_2)$ can be evaluated from

$$F_1 = k\left[\frac{(i_b+i_x)^2}{(g_0-\bar{x})^2}-\frac{(i_b-i_x)^2}{(g_0+\bar{x})^2}\right]$$

$$\text{where } \bar{x}=0,\ i_x \equiv i_b=0.5\,A,\ i_b=0.5\,A,\ g_0=0.001\,m$$

$$F_1 = 10^{-4}\left[\frac{1}{0.001^2}-0\right] \quad (8)$$

$$=100\,N$$

The i_x arises from output membership function I1 as can be seen in figure 7b resulting in F_1 = 100N, shown in figure 12a. Similar from (5), for $-i_x$ the force, F_2 = -100N.

The rules used to reset the rotor in equilibrium from the initial position $x(0)$ = 0.0008m, can be found from the rule Matrix Table 2. For +ve $x(0)$, the command $-i_x$ is required. Which will create net –ve force F_x to pull back the rotor to the left, as can be seen in figure 2. The magnitude of the F_x is then

$$F_x = k\left[\frac{(i_b + i_x)^2}{(g_0 - x)^2} - \frac{(i_b - i_x)^2}{(g_0 + x)^2}\right]$$

$$\text{where } x(0) = 0.0008\,m,\ -i_x = -0.5A,\ F_1 = 0\,N \qquad (9)$$

$$F_x = 10^{-4}\left[0 - \frac{(0.5 - (-0.5))^2}{(0.001 + 0.0008)^2}\right]$$

$$= -30.86\,N$$

The initial force F_x can be seen in figure 12 at $x(0)$.

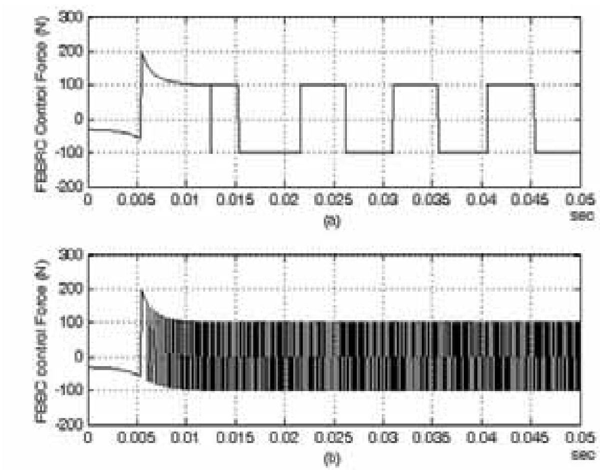

Fig.12. a. Low frequency control force output from FBBRC (b) High frequency control output from FBBC

V. Conclusions

A new fuzzy bang-bang relay controller (FBBRC) is proposed in this paper to control the active magnetic bearing system. The proposed controller combines fuzzy logic intelligence and hard limiting relay into one entity. The controller is inherently optimal in time due to its bang-bang action. The configuration of the new controller is simpler than conventional fuzzy controller. It uses only two choices in output membership function resulting in a simple rule matrix Comparison with other conventional fuzzy bang-bang (FBBC) technique reveals that transient response time is closely same or better for FBBRC. However, the steady state response of FBBC is unstable due to high frequency components, which can cause instability and distortion in the current amplifiers. Proportional derivative controller was also compared, but due to its fixed parameter selection could not match the performance of the fuzzy controllers. To determine the PD parameters for a particular initial condition, a simple optimization simulation method is also presented.

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Nomenclature

$\widetilde{A}$ fuzzy input set

$\widetilde{A}_k^j$ j^{th} linguistic value of input linguistic variable $\widetilde{x}_k$

A_g Area of one magnetic pole, m^2

$\widetilde{B}$ Fuzzy output set

$\widetilde{B}_l^j$ Linguistic values for output linguistic variable $\widetilde{y}_k$

B Magnetic flux density, Wb

B_b Bias magnetic flux density, Wb

B_p Perturbation magnetic flux density, Wb

c fuzzy rule index

C number of fuzzy rules

D Derivative feedback gain, As/m

F Magnetic actuator force, N

F_1 Magnetic force of the actuator located at the positive X -axis, N

F_2 Magnetic force of the actuator located at the negative X -axis, Y

F_x Resultant force of the magnetic actuator in the X -direction, N

F_o open loop magnetic force

g_0 Nominal air gap length, m

H Magnetic field intensity, At/m

I1 decomposition of linguistic rules $\mu_{\widetilde{B}_1^2}$ - on/off command

I2 decomposition of linguistic rules $\mu_{\widetilde{B}_1^1}$ -on/off command

i Magnetic coil current, A

i_1 Magnetic coil current in the actuator located at the positive X -axis, A

i_2 Magnetic coil current in the actuator located at the negative X -axis, A

i_b Bias magnetic coil current, A

i_p Perturbation magnetic coil current, A

i_x Perturbation magnetic coil current in the actuator located at the positive X -axis, A

j linguistic value index

k input variable index to fuzzy controller

l Length of flux path, m

l_{fe} Mean length of iron, m

m Mass of rotor, kg

N Number of coil turns

P Proportional feedback gain, A/m

Φ Magnetic flux, Wb

R Magnetic circuit reluctance, At/Wb

r index fuzzy rules

x	Rotor displacement in the X -direction, m
x_k	fuzzy inputs
$\widetilde{x}_k$	Linguistic input variable
$\dot{x}$	Rotor velocity in the X -direction, m/s
$\ddot{x}$	Rotor acceleration in the X -direction, m/s^2
μ_0	Permeability of free space, $4\pi\times 10^{-7}$ H/m
μ_r	Relative permeability of medium in magnetic field
$\mu_{\widetilde{A}_k^j}$	Decomposition of fuzzy input variable
$\mu_{\widetilde{B}_k^j}$	Decomposition of fuzzy output variable
$\mu_{overall}$	Decomposition of overall implied fuzzy rules
X_k	input universe of discourse
$\boldsymbol{Y_{bb}}$	output universe of discourse for fuzzy bang-bang controller
$\boldsymbol{Y_{FLC}}$	output universe of discourse for conventional fuzzy controller
y	defuzzified output
$\widetilde{y}_i$	linguistic output variable
y^{crisp}	defuzzified crisp output